Data-driven Inverse Optimization with Application to Dynamic Line Rating in Russian Power Grid

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***Abstract*—Dynamic line rating technology has been evolving since the late nineteen seventies. However, it is not used to its full extend. This study presents a methodology for calculating the benefits of dynamic line rating in the Russian power grid. The economic benefits of using a dynamic line rating instead of a conventional static line rating are estimated for a simple but representative two-zone network representing a multi-unit Russian energy corridor connecting the Siberian and European zones. Due to the lack of precise information on the generation units' costs and capacity, we present an inverse optimization model to infer these parameters from the available electricity price and aggregated consumption. Our simulations show that using dynamic line rating versus static rating results in savings of up to 1.65% per day.**

***Keywords*—Dynamic Line Rating, Inverse Optimization, Russian Power System**

I. Introduction

A. Motivation

Due to various historical events, the Russian power industry has remained on the sidelines for updates over the past 30 years. Most of the existing power plants date back to the Soviet Union days. They will need to be decommissioned, modernized, or replaced with new ones during 2025-2035. Rapid development and growth of Russia's regions is expected soon, requiring additional power supply, both by power capacity and transmission lines installation [1]. By 2035, forecasts show an increase in electricity demand by 35-47 GW. It has been reported that by 2035 there may be a shortage of generating capacities of 54-66 GW [2].

The construction of new power lines and the associated costs can become a challenge from a practical point of view due to the area's natural features, e.g., swamps and forests. The use of dynamic line rating (DLR) in Russia should help reducing capacity needs. DLR utilizes meteorological information to dynamically update transmission lines' power capacity instead of utilizing fixed conservative ratings. The combination of cold winters coupled with high electricity demand, and long power transmission corridors, makes DLR an exciting solution for delaying transmission investments in the Russian federation.

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In this paper, we analyze the impact of *non-wire alternatives* solution, particularly the dynamic line rating solution, in the energy corridor connecting the Siberian region and the European part of the Russian Federation.

B. Literature Review

Society's development is generally accompanied by increase in demand that would require the upgrade of electricity networks [3]. In practice, there are two options for solving capacity adequacy problems associated with the upgrades of the power grid. First, it is possible to deploy new power lines or change the conductor diameter of existing lines. The second option is the use of non-wire alternative technologies, for example, dynamic line rating, demand response, local generation, among others [4].

The dynamic line rating is based on the fact that the amount of current that is carried through the power line can be changed over time depending on the weather conditions. This allows the grid operator to automatically update the lines' current capacity, provided that weather conditions are monitored in real time and used appropriately [5]. Today, in Russia, most of the limits for power lines are set at fixed values for the permissible current load. This practice is well known as static line rating (SLR). The SLR is usually calculated for each region separately based on the worst-case historical and meteorological data [6].

The maximum operating flow through the transmission line when using SLR in Russia is 130% of the rated capacity [6]. There is ample evidence in electric power practice that many transmission lines can operate up to 130% of the SLR for 90% of the year (Fig. 1). If this capacity is used through DLR, then it becomes possible to serve more customers using the existing grid by increasing the line limits when the weather conditions allow it. At the same time, there is no need for the construction of new power lines and facilities associated with them.

Electricity prices in the Siberian zone are significantly lower than in the European zone of Russia. This difference can be explained by the fact that Siberia has a large number of cheaper energy resources. Hydroelectric power plants predominate in the generation mix, using energy from river flows to generate electricity. At the same time, in the European part of Russia there are more fuel-based and nuclear power plants for which more expensive primary energy sources are used. Therefore, it becomes attractive the analysis of the DLR use in

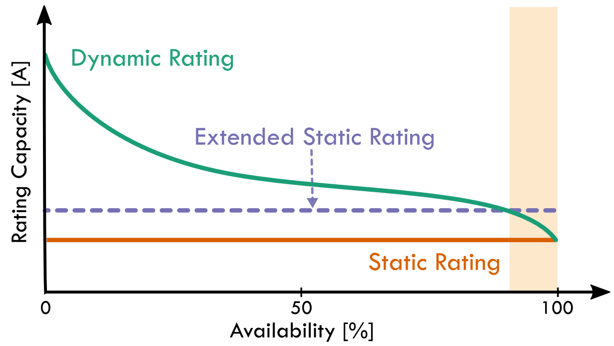

Figure 1: Distribution of dynamic and static line ratings in time

the transmission corridor between the Siberian and European zones, since it could provide economic benefits for the system. However, access to exact data related to the energy sector in Russia is mainly confidential, for instance, generating costs are not publicly available while zonal prices are. In order to make a good assessment for the DLR impact it is then necessary to infer the generation cost and capacity parameters from the publicly available data. For doing so, we will rely on *inverse optimization techniques*.

Inverse optimization problems relate to the fitting or learning problem where the parametric function is another optimization problem with fully or partially unknown parameters. Recent work [7] has demonstrated the capabilities to derive demand utility parameters through the use of price and consumption data.

C. Paper Contributions and Organization

The contributions of this paper are twofold.

- The first contribution corresponds to the methodology itself. We propose a learning model based on inverse optimization to find the cost coefficients and generating capacities, using data available from the electricity prices and total consumption [8].
- The second contribution lays in testing our methodology on a real case. Dynamic line rating application to the Russian power grid corridor between Siberia and European part of Russia is tested to quantify the overall cost reduction.

The rest of the paper is organized as follows. First, we provide an overview on the dynamic line rating technology and technical solutions for its implementation in section II. Next, we present mathematical modeling for the OPF with DLR and the inverse optimization problem in section III. In section IV a case study is implemented for the Russian electrical corridor between Siberian and European zones. Finally, the conclusions are summarized in section V.

II. Dynamic Line Rating

Dynamic line rating accounts the following physical phenomena: i) the Joule effect, i.e., current flow in the conductor is accompanied by heat gain [9]; ii) the solar radiation that increases the temperature of a conductor due to heat absorption [9]; iii) natural and forced convection; iv) black-body radiation; v) transmission line sagging, i.e., an increase of temperature results in conductor expansion [10].

A. Technical Solutions for Dynamic Line Rating

There are two methods for tracking line and climatic parameters [11]. The first one relies on sensors to be installed along the entire power line at each end of it. Another way is to exclusively use the analysis of weather data such as air temperature, wind speed, and solar radiation collected by meteorological services. When these weather parameters are obtained, it becomes possible to apply the thermal line model to calculate the maximum allowable current.

The method of direct measurements from sensors is preferable for obtaining data from power lines in real time. The sensors are able to measure temperature, line sag and wind speed by displacing the wire from its initial position [11]. However, due to the distances of several kilometers over which the power lines are stretched, this method can become prohibitively expensive due to the large number of required sensors. Therefore, the use of meteorological data from weather stations is used as a good and fairly cheap approximation to the necessary information.

B. General Model Representation of DLR

The maximum current-carrying capacity of an overhead transmission line is described in CIGRE and IEEE standards [12], [13]. Both technical reports provide DLR methodology calculation based on the heat balance equation (1):

$$P_c + P_r = P_j + P_s \tag{1}$$

where P_c is the convective cooling, P_r is the radiation cooling, P_j is the heat gained through Joule effect, and P_s is the solar heat gain.

Since the power line is heated by the Joule effect and is in the air, there is a local temperature gradient in the area next to the wire. The calculation is made using the thermal conductivity of the air and the Nusselt number [14].

Solar radiation directly affects the transfer of energy through wires, as it causes the temperature of the conductor to rise. Consequently, the increase in solar radiation will reduce the dynamic rating of the transmission line.

Finally, the heat released through the Joule effect is calculated using the square of the amperage multiplied by the value of resistance of the wire, and the radiation cooling is calculated using the Stefan-Boltzmann law.

III. Mathematical Modeling

A. Optimal Power Flow with Dynamic Line Rating

To quantify the possible economic impact of changes in the capacity of DLR-based links, a two-node system representing the European and Siberian zones is. In our case, the simulation was carried out with a two-node grid for convenience, but if necessary, it can be easily scaled to any grid size.

The economic results of using DLR instead of SLR can be obtained by adjusting the transmission capacity between the considered zones on the optimal power flow (OPF) problem.

The OPF problem is described in (2). The objective function of this problem is to minimize the total aggregated system costs. Generation costs are assumed to be a function of the energy produced, and are calculated based on bid costs c_b^Z representing generation capacity blocks b and zone $Z = \{E, S\}$. The generation bid costs c_b^Z are assumed constant during the studied time horizon. The load balance in the Siberian zone ($Z = S$) is described in (2b), while (2c) describes the European zone ($Z = E$). The energy flow from the Siberian to the European zone is represented as f_t. The transmission limit is set by the formula (2d), where F_t is the transmission capacity. The generation limit in the Siberian and European zones is represented by (2e) and (2f), respectively.

MODEL Optimal Power Flow between the European and Siberian zones considering Dynamic Line Rating

Objective:

$$\min_{x_b^S(t), x_b^E(t), f(t)} \sum_b \sum_t c_b^S x_b^S(t) + c_b^E x_b^E(t) \tag{2a}$$

Constraints:

Siberian balance: $$\sum_b x_b^S(t) - f(t) = d^S(t) \tag{2b}$$

European balance: $$\sum_b x_b^E(t) + f(t) = d^E(t) \tag{2c}$$

Line capacity: $$-F^{\text{DLR}}(t) \le f(t) \le F^{\text{DLR}}(t) \tag{2d}$$

Sib. generation limit: $$0 \le x_b^S(t) \le X_b^S \tag{2e}$$

Eur. generation limit: $$0 \le x_b^E(t) \le X_b^E \tag{2f}$$

At this point, it is essential to highlight that line loadability can be determined by three main issues, namely (i) thermal limit, (ii) voltage-drop limit, and (iii) steady-state stability limit. Dynamic line rating links with the first one, the thermal limit, which is quite common in most of the transmission lines [15]. However, when transmission lines are larger than 100 km, the voltage-drop limits (up to 300 km) or the steady-state stability limit (more than 300 km) determine loadability. Both voltage-drop and steady-state stability limits can be incorporated in the OPF model by Kirchhoff voltage's Law and a bounding constraint on the voltage phase angle difference, respectively.

B. Market Parameters Inference

To properly formulate problem (2) it is necessary to know the equivalent generation parameters c_b^Z and X_b^Z for each of the zones. For this purpose, we present in this section the inverse optimization model used to derive the generation parameter from historical price and demand observations [8].

Given an observation i representing one hour, $[p^S(i), p^E(i), \lambda^S(i), \lambda^E(i)]^T$ resulting from solution of an OPF for such observation, i.e., solving the $\mathcal{P}(i)$.

$$\mathcal{P}(i): \min_{x_b^S(i), x_b^E(i), f(i)} \sum_b c_b^S x_b^S(i) + c_b^E x_b^E(i) \tag{3a}$$

subject to:

$$\sum_b x_b^S(i) - f(i) = d^S(i), \quad : \lambda^S(i) \tag{3b}$$

$$\sum_b x_b^E(i) + f(i) = d^E(i), \quad : \lambda^E(i) \tag{3c}$$

$$-F \le f(i) \le F, \quad : \underline{\mu}(i), \overline{\mu}(i) \tag{3d}$$

$$0 \le x_b^S(i) \le X_b^S, \quad \forall b \quad : \underline{\psi}_b^S(i), \overline{\psi}_b^S(i) \tag{3e}$$

$$0 \le x_b^E(i) \le X_b^E, \quad \forall b \quad : \underline{\psi}_b^E(i), \overline{\psi}_b^E(i) \tag{3f}$$

Problem (3) is a linear programming (LP) model. For the simplicity of our model we assume that line capacity, F, generating cost, c_b^E, c_b^S, and capacity X_b^E, X_b^S of each block does not change, but the other variables and parameters change (see dependence on i).

For this problem, we can derive the first-order optimal conditions, Karush-Kuhn-Tucker (KKT) optimality conditions [16], [17]. The KKT conditions for problem $\mathcal{P}(i)$ are represented by the following set of constraints:

Primal feasibility:

$$p^S(i) = \sum_b x_b^S(i), \quad : \beta^S(i) \tag{4a}$$

$$p^E(i) = \sum_b x_b^E(i), \quad : \beta^E(i) \tag{4b}$$

$$p^S(i) - f(i) = d^S(i), \quad : \lambda^S(i) \tag{4c}$$

$$p^E(i) + f(i) = d^E(i), \quad : \lambda^E(i) \tag{4d}$$

$$-F \le f(i) \le F \tag{4e}$$

$$0 \le x_b^S(i) \le X_b^S, \quad : \forall b \tag{4f}$$

$$0 \le x_b^E(i) \le X_b^E, \quad : \forall b \tag{4g}$$

Dual feasibility:

$$\frac{\partial L}{\partial x_b^S}: \quad \lambda^S(i) = c_b^S + \overline{\psi}_b^S(i) - \underline{\psi}_b^S(i), \quad : \forall b \tag{5a}$$

$$\frac{\partial L}{\partial x_b^E}: \quad \lambda^E(i) = c_b^E + \overline{\psi}_b^E(i) - \underline{\psi}_b^E(i), \quad : \forall b \tag{5b}$$

$$\frac{\partial L}{\partial f}: \quad -\lambda^S(i) + \lambda^E(i) + \overline{\mu}(i) - \underline{\mu}(i) = 0 \tag{5c}$$

$$\overline{\mu}(i), \underline{\mu}(i) \ge 0 \tag{5d}$$

$$\overline{\psi}_b^S(i), \underline{\psi}_b^S(i), \overline{\psi}_b^E(i), \underline{\psi}_b^E(i) \ge 0 \quad : \forall b \tag{5e}$$

Complementary conditions:

$$\underline{\mu}(i)[F - f(i)] = 0 \tag{6a}$$

$$\overline{\mu}(i)[f(i) - F] = 0 \tag{6b}$$

$$\overline{\psi}_b^S(i)[x_b^S(i) - X_b^S] = 0, \quad : \forall b \tag{6c}$$

$$\underline{\psi}_b^S(i)[0 - x_b^S(i)] = 0, \quad : \forall b \tag{6d}$$

$$\overline{\psi}_b^E(i)[x_b^E(i) - X_b^E] = 0, \quad : \forall b \tag{6e}$$

$$\underline{\psi}_b^E(i)[0 - x_b^E(i)] = 0, \quad : \forall b \tag{6f}$$

We would like to estimate the parameters of the model given N observations while ensuring consistency with observed data and the OPF problem. For doing so, we propose a least absolute value (LAV) model (7) with structural constraints of the first-order optimally conditions.

Model LAV Inverse Optimization. Learning Model

Objective:

$$\min_{\Omega} \; \alpha_1 \sum_{i=1}^{N} |\lambda_i^S(i) - \hat{\lambda}_i^S(i)| + \alpha_2 \sum_{i=1}^{N} |\lambda_i^E(i) - \hat{\lambda}_i^E(i)| \quad (7a)$$

Constraints:

$$\textit{Primal feasibility } (4) \qquad \forall i = 1 \dots N \quad (7b)$$

$$\textit{Dual feasibility } (5) \qquad \forall i = 1 \dots N \quad (7c)$$

$$\textit{Complementary conditions } (6) \qquad \forall i = 1 \dots N \quad (7d)$$

$$0 \leq c_b^S \leq c_{b+1}^S, \quad \forall b = 1 : B-1 \quad (7e)$$

$$0 \leq c_b^E \leq c_{b+1}^E, \quad \forall b = 1 : B-1 \quad (7f)$$

$$F, X_b^E, X_b^S \geq 0 \quad (7g)$$

where $\Omega = \{p_i^S(i), p_i^E(i), \lambda_i^S(i), \lambda_i^E(i), x_b^S(i), x_b^E(i), f(i), c_b^S, c_b^E, F, X_b^S, X_b^E\}$

In (7) the objective function (7a) contains two weighted terms that accumulate the absolutes error value between the observed electricity price and the estimated one for both, Siberian and European zones. Problem (7) is a non-linear optimization problem. We have transformed (7) into an equivalent problem where the absolute value operator is linearized and the complementary conditions regularized in the objective function. The equivalent problem is non-linear with a linear feasible set. When solving this problem, it is possible to check if the obtained solution corresponds to the global optimum by checking if complementary is satisfied.

IV. Case Study

In this section, we first present the data that was taken from the wholesale energy market of the Russian Federation. Further, using this data, we obtain the necessary market parameters through the formulated inverse optimization problem. The parameters are then applied to the OPF with DLR problem in Russia and the results are described. The presented calculations were performed in Julia 1.5 with the optimization suite JuMP [18], [19].

A. System Data

For the simulations the real data from the wholesale electricity market from November 1, 2010 to October 31, 2011 was used [8]. The data on the day-ahead electricity price and demand obtained for the indicated periods for both zones are respectively presented in Fig. 2 and Fig. 3. One can note that prices in the Siberian zone are much lower than prices in the European zone, which is explained by the large installation of hydropower plants in the Siberian zone. In both zones, the seasonality of electricity demand is approximately the same with a noticeable increase in winter.

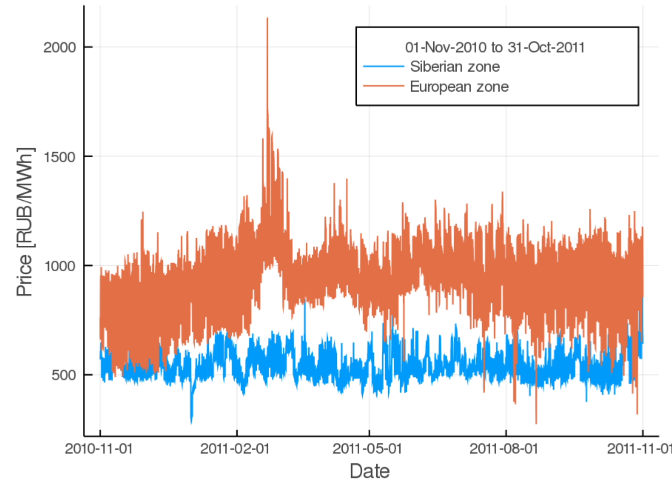

Figure 2: Electricity prices for the selected year.

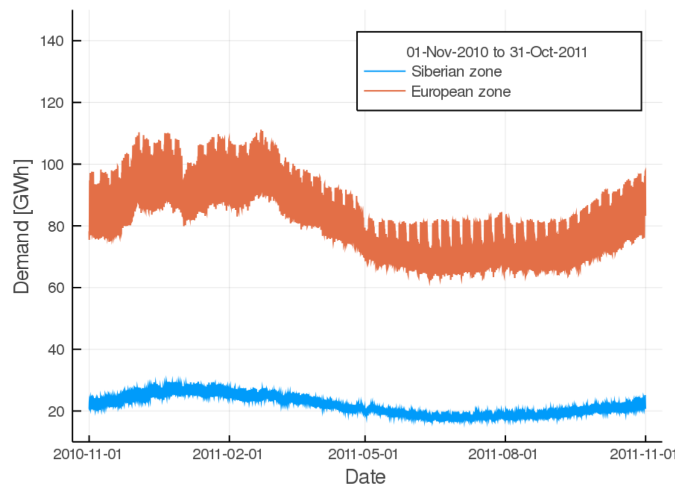

Figure 3: Power demand for 1 year.

Due to the lack of access to real data for the transmission corridor between the European and Siberian zones, we considered a single 110 kV line. The modeled line consists of aluminum wires with a cross-sectional area of 70 mm^2 with a maximum operating current of 265 A and a maximum operating temperature of 70°C [20]. The emissivity for aluminum wire is 0.2 [21]. The selected testing region is Novosibirsk, for which reference [22] provides the meteorological data.

Fig. 4 shows the calculated average line rating after using expression (1) to the above-mentioned weather and line data (September 1-11, 2011). We observe that the average DLR is different for every day. In particular, the obtained DLR is significantly higher, at least 1.6 times, than the SLR.

B. Market Parameter Learning Results

Parameters for the DLR-based model (2) are estimated using the inverse optimization model (7) and input data observed

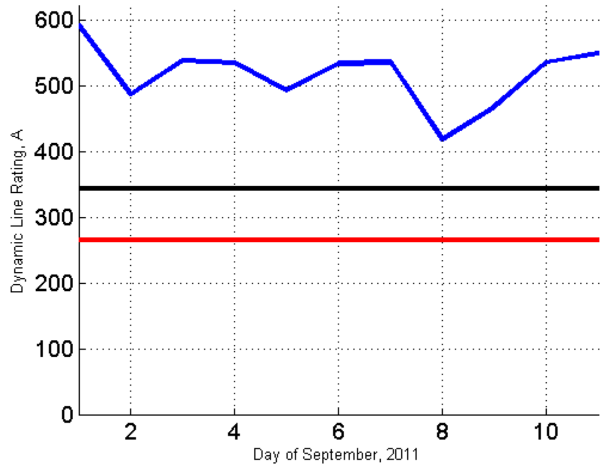

Figure 4: Line ratings, wind speed as a cooling factor was accounted. Blue line represents dynamic line rating (DLR) of considered transmission line; red line represents static line rating (SLR) and black line is for 130% of static rated capacity.

from the market [8]. We run the inverse model with 2 different datasets.

First, we used 250 samples for the learning process (September 1-5, 2011). In this model, we have also artificially divided each node into 5 blocks. The simulation results are shown in Fig. 5, where the red dots on the graphs represent the realizations of the energy generation cost and the blue lines the inferred cost curves. In the European zone the generation cost grows gradually, while in Siberia there is a sharp jump for the inclusion of the last block. However prices in Siberia are much lower than in the European part of Russia. This fact reflects the differences in the zonal energy mix.

To compare the effect of the sample size on the inferred costs curves, we ran the learning model with a set with 100

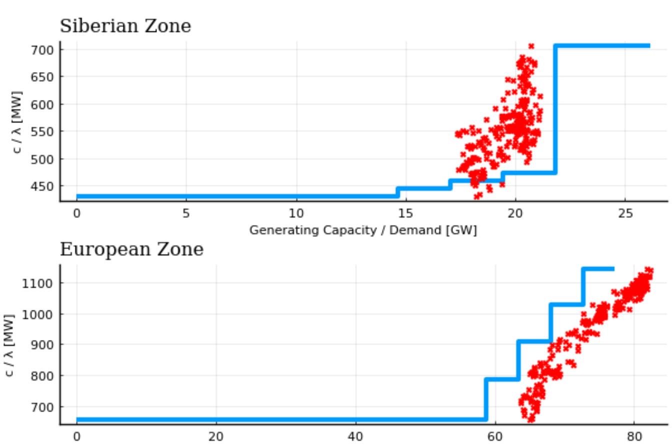

Figure 5: The results of calculating market parameters for the period September 1 – September 11, 2011 (250 samples in total).

samples. The summary of the obtained cost coefficients and capacities is presented in Table I. The upper part of the table shows the costs for each sequentially connected generation block, and the lower part of the table shows the capacity associated with each block. The increase in code execution time occurred disproportionately, from 100 to 250 samples, with 270 against 1 542 seconds. Comparing the results for 100 and 250 samples, we can say that the numerical indicators of cost coefficients and power capacities produced in each of the blocks do not differ greatly from each other. In further calculations and plots, the dataset with 250 samples was used in order to track the dynamics of processes more clearly.

C. OPF with DLR

Once the values of the market parameters have been obtained after the learning process, we can simulate the behavior of the system in the cases of SLR and DLR.

The use of DLR leads to an increase in the amount of power that is transmitted from the Siberian zone to the European zone, Fig. 6. This additional energy transfer is due to the increase in generation from the less expensive Siberian, Fig. 7. Also for an integrated system, when DLR is used instead of SLR, the cost is decreased in a pattern that emulates the DLR capacity changes, Fig. 8

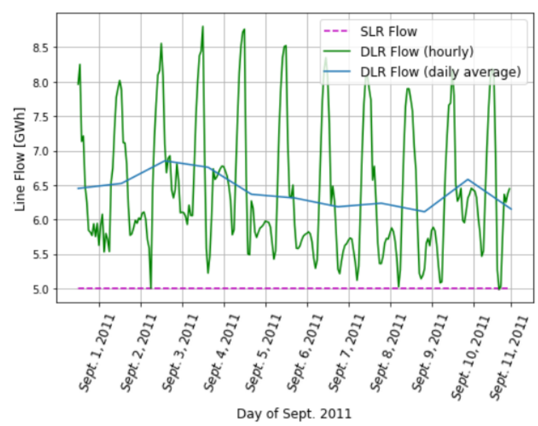

Figure 6: Line flow from the Siberian to the European zone.

V. Conclusions

This paper describes a technical and economic analysis of the potential benefits that can be obtained in the Russian power system by using dynamic line rating.

A methodology for calculating the system generation cost and capacity paramaters based on an inverse optimization model and data on the demand and prices for electricity is presented and tested. The technical results are then applied to a model of the optimal power flow between the European and Siberian parts of the Russian wholesale electricity market where the effects of dynamic line rating are analyzed. Using a dynamic rating instead of a static one leads to cost savings in the system operation, increasing efficiency up to 1.65%.

Further research directions are associated with an increase in the time resolution (a period with a larger number of days for

TABLE I. LEARNING MARKET PARAMETERS RESULTS

Input vector size	c_b^S, [Rubles/MW]					c_b^E, [Rubles/MW]				
	block 1	**block 2**	**block 3**	**block 4**	**block 5**	**block 1**	**block 2**	**block 3**	**block 4**	**block 5**
100	431.08	443.839	456.599	469.358	686.27	671.67	789.024	901.44	1011.019	1119.29
250	431.08	444.792	458.505	472.217	705.33	656.84	785.022	907.213	1025.665	1141.85
Input vector size	X_b^S, [MW]					X_b^E, [MW]				
	block 1	**block 2**	**block 3**	**block 4**	**block 5**	**block 1**	**block 2**	**block 3**	**block 4**	**block 5**
100	16982.288	1471.147	1471.563	2075.105	4144.685	59041.771	4530.866	4530.771	4530.844	4530.747
250	14659.932	2386.023	2385.949	2384.605	4328.453	58787.084	4620.979	4620.979	4620.979	4620.979

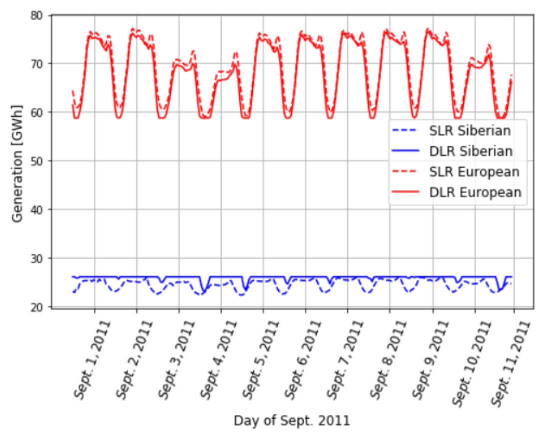

Figure 7: Zonal generation for both SLR and DLR.

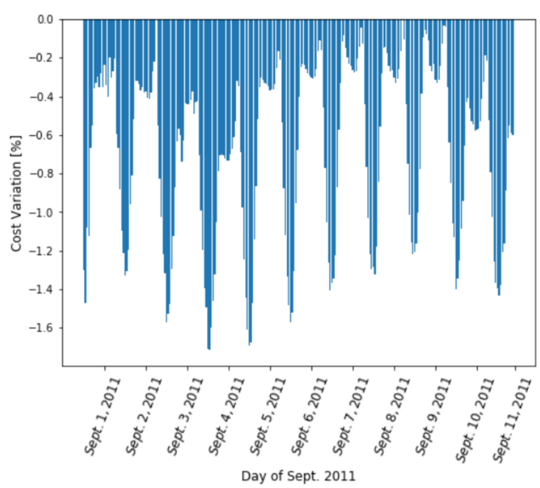

Figure 8: Cost variation when implementing DLR, instead of SLR, between the two zones.

the learning process and efficiency assessment), an extension of the case under consideration to a large power grid.

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