An Optimal Scenario Reduction Method for Stochastic Power System Problems

Alvaro Gonzalez-Castellanos and David Pozo
Center for Energy Science and Technology
Skolkovo Institute of Science and Technology (*Skoltech*), Moscow, Russia
alvaro.gonzalez@skolkovotech.ru, d.pozo@skoltech.ru

***Abstract*—Modern power systems problems recognize the value of using stochastic information to model renewable generation operations properly. Many optimal operation and planning problems can be formulated as two-stage stochastic optimization problems, typically employing discrete probabilistic scenarios to describe stochastic information. Large discrete scenario sets increase the computational complexity of these problems. Therefore, it is essential scenario reduction techniques that provide a good representative scenario set yielding the same or similar optimal results as the original probabilistic set.**

This paper describes a generalized adaptive partition-based method (GAPM) for finding an optimal representative scenario set for two-stage stochastic optimization problems. The iterative methodology obtains the optimal partition for a scenario set while providing the optimality gap at each iteration. Numerical tests are performed to compare the GAPM with popular scenario reduction techniques employed in the power systems literature. We show that even for a simple small problem, popular scenario reduction techniques based on the Kantorovich distance exhibit optimally gaps of more than 5%, performing even worse than vanilla methods of scenario reduction. Our methodology, in principle, can guarantee any optimality gap between the original problem and the reduced one.

***Index Terms*—Stochastic Optimization, Scenario Reduction, Renewable Generation**

NOMENCLATURE

Most of the notation of this paper is introduced throughout the text. Regarding the probabilistic uncertainty description, the symbol ξ represents a vector of random variables. For simplicity, the source of uncertainty used in this work is a two-dimensional vector capturing demand and renewable generation. Realizations or scenarios of ξ, are represented by ξ^ω or, in short as $\omega \in \Omega$ for convenience. Ω is the sample space set of ξ (either continuous or discrete). P refers to a partition of the sample space Ω, thus $P \subseteq \Omega$. A collection of partitions P of the sample space Ω is denoted by $\mathcal{P}$. The symbol $\mathbb{E}[\nu]$ denotes the mathematical expectation of a random variable ν, while $\mathbb{E}[\nu|P]$ represents the conditional mathematical expectation of ν on the partition P. Probability of an scenario ω is represented by $\mathbb{P}(\omega)$, or alternative by $\rho(\omega)$, while the probability of a partition P is given by $\mathbb{P}(P)$.

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I. INTRODUCTION

The rapid integration of renewable energy sources in power systems, supported by technological and political developments, requires introducing stochastic information in operation and planning decision-making processes [1]. A common approach for the characterization of stochastic information in power system models is to employ discrete probabilistic scenarios based on historical or forecasted data. The discrete probabilistic scenarios can be generated using auto-regressive models [2], or more sophisticated hybrid methods [3]. Once the discrete scenario set that characterizes the stochastic information has been defined, the stochastic optimization problem with fixed recourse can be represented as the following equivalent two-stage stochastic program:

$$z = \min_{x \in \mathcal{X}}. \{c^\top x + \mathbb{E}[\mathcal{Q}(x, \xi)]\} \tag{1}$$

where

$$\mathcal{Q}(x, \xi) = \min_{y \geq 0}. \{q^\top y | Wy = h^\xi - T^\xi x\} \tag{2}$$

The standard approach for solving problem (1) is to represent the random vector ξ by drawing finite set of realizations (scenarios) indexed by ω, with an associated probability $\mathbb{P}(\omega)$ [4]. Then, the expectation term can be rewritten as:

$$\mathbb{E}[\mathcal{Q}(x, \xi)] = \sum_{\omega \in \Omega} \mathbb{P}(\omega) \mathcal{Q}(x, \xi^\omega) \tag{3}$$

A sufficiently large number of scenarios is desired to represent the stochastic nature of the optimization problem properly while inferring optimal or near-optimal first-stage decisions on the original problem (1). However, in practice, the number of discrete scenarios in the set Ω must be kept relatively low to guarantee the two-stage stochastic programs' computational tractability. Therefore, there exists a need to reduce the number of discrete scenarios employed while preserving the problem's stochastic information. This procedure is called scenario reduction and there have been proposed several approaches in the literature [5]. We refer to some of the prominent works applied to power systems problems in the subsequent sections.

II. SCENARIO REDUCTION TECHNIQUES

To identify the optimal partitions of an uncertainty scenario set Ω, the most common approach is to group scenarios into a representative set $\Omega^{\mathrm{R}}\subset\Omega$ that recovers the features of the original set Ω. The representative set is said to be close to the original problem if there is a sufficiently small distance between the scenario sets of the reduced and original problems [6]. The Kantorovich distance, $D_K(Q,Q')$, is commonly employed to determine the probabilistic distance between two scenario sets Ω and Ω' with probability functions Q and Q', respectively. Thus, the Kantorovich distance could be employed for determining the optimal reduced scenario set Ω^{R}. $D_K(Q,Q')$ is obtained by solving the Monge-Kantorovich mass transportation problem for discrete scenario sets [7]:

$$D_K(Q,Q') = \min_{\rho(\omega,\omega')} \Bigg\{ \sum_{\omega,\omega'} c(\omega,\omega')\rho(\omega,\omega') : \sum_{\omega} \rho(\omega,\omega') = \pi_{\omega'} \quad \sum_{\omega'} \rho(\omega,\omega') = \pi_{\omega} \Bigg\}, \tag{4}$$

where π_ω and $\pi_{\omega'}$ are the scenario probabilities in their respective sets. $\rho(\omega,\omega')$ is the joint probability defined over $Q\times Q'$. The function $c(\omega,\omega')$, known as the cost function, is continuous, nonnegative, and symmetric. Common choices for the cost function $c(\cdot)$ are norms on $\mathbb{R}^n$.

Problem (4) could be computationally challenging for sets with large number of scenarios. To overcame this issue, a forward selection algorithm can be employed as a heuristic procedure to determine the composition of a representative set with a given cardinality [6]. In this algorithm, the reduced set is filled by iteratively adding the scenario that minimizes the Kantorovich distance between the reduced set Ω^{R} and that of the remaining scenarios, $\Omega_J=\Omega\setminus\Omega^{\mathrm{R}}$.

A. Forward Selection Algorithm

The pioneer work of Dupačová *et al.* [6] introduced a *forward selection algorithm* for scenario reduction in stochastic optimization problems with discrete probability distributions.The forward selection algorithm is an heuristic for solving the problem (4).

The steps of the **forward selection algorithm** are:

Step 0) Scenario pre-processing. Calculate the $c(\omega,\omega')$ value for each scenario pair in the original scenario set $\omega,\omega'\in\Omega$. Initiate the empty representative set $\Omega^R=\varnothing$ and the set of non-selected scenarios $\Omega_J^{[0]}=\{1,...,N_\Omega\}$.

Step 1) Select the scenarios to be added to the representative set. In an iterative manner, identify the scenarios $\omega_i\in\Omega$, such that at iteration i:

$$\omega_i \in \arg\min_{\omega\in\Omega\setminus\Omega^{\mathrm{R}}} d_\omega \tag{5}$$

where

$$d_\omega = \sum_{\omega'\in\Omega_J^{[i-1]}\setminus\omega} \pi_\omega c(\omega',\omega) \tag{6}$$

In each iteration i, ω_i is added to the representative set Ω^{R} and removed from the non-selected set $\Omega_J^{[i]}=\Omega_J^{[i-1]}\setminus\omega_i$.

At the beginning of each new iteration i, the cost function between non-selected scenarios $\omega,\omega'\in\Omega_J^{[i-1]}$ is updated by:

$$c^{[i]}(\omega,\omega') = \min\Big\{c^{[i-1]}(\omega,\omega'), c^{[i-1]}(\omega,\omega_{i-1})\Big\} \tag{7}$$

Step 2) Stopping criteria. This iterative greedy algorithm stops once the desired number of scenarios N^{R}, in the reduced set, is reached or when the Kantorovich distance between the original and reduced set is below a threshold.

Step 3) Update the representative scenarios' probability. The new assigned probability for the representative scenarios is calculated as [8]:

$$\pi^*_{\omega'} \leftarrow \pi_{\omega'} + \sum_{\omega\in J(\omega')} \pi_\omega, \forall\omega'\in\Omega' \tag{8}$$

where

$$J(\omega')=\{\omega\in\Omega_J|\omega'=j(\omega)\},\;\; j(\omega)\in\arg\min_{\omega'\in\Omega^{\mathrm{R}}} c(\omega',\omega) \tag{9}$$

The choice of the cost function $c(\cdot)$ influences the scenario selection. Three popular scenario reduction techniques (SRTs) with their cost functions are presented in the next subsections.

B. Norm of the Difference Between Pairs of Random Vectors:

Dupačová *et al.* [6] introduced the forward selection algorithm depicted in the previous subsection. The proposed approach focuses on reducing the scenario set based on the random input parameters, e.g., renewable generation or electric demand. In this sense, there is no contextual information about the problem where the reduced scenario set should be applied. The cost function is defined as:

$$c(\omega,\omega') = \|h^\omega - h^{\omega'}\| \tag{10}$$

In this manner, the scenario reduction procedure would find representative scenarios based on how similar is the behavior of the stochastic parameter h^ω among scenarios. Thus, preserving the original set's stochastic information.

C. Difference between Single-Scenario Objectives with Fixed First Stage:

The methodology proposed by Dupačová *et al.* guarantees that the reduced scenario set being statistically close to the original scenario set. However, this method does not guarantee that the reduced set's second-stage decisions will be close to those of the original one. Therefore, it can be said that the SRT proposed by Dupačová *et al.* is based on the nature of the stochastic parameters, rather than that of the stochastic problem itself. To overcome this issue, Morales *et al.* [9], presented a modified cost function based on the objective function of the deterministic expected-value problem (DP), i.e., the random parameters are replaced by their expectation. The cost function is given by:

$$c(\omega,\omega') = \|z^{\mathrm{DP}}_\omega - z^{\mathrm{DP}}_{\omega'}\|, \tag{11}$$

where z^{DP}_ω is the objective value of the stochastic problem if the second stage is represented solely by scenario ω and the first stage is fixed at the DP solution. Note that in this methodology, firstly, the DP problem must be solved, and then z^{DP}_ω is calculated for every scenario ω in the original set Ω.

D. Difference between Disjoint Single-Scenario Objectives:

The approach employed by Morales *et al.* might yield risk–averse solutions, since in the initial DP, the optimal solution will avoid making use of highly-expensive load shedding mechanisms. Thus, resulting in generation over-scheduling that does not consider the possible contributions of higher-than-average renewable generation scenarios. Bruninx *et al.* presented a risk-neutral SRT based on the forward selection algorithm [10]. This SRT's cost function is given by:

$$c(\omega,\omega') = \|z_{\omega}^{\text{SS}} - z_{\omega'}^{\text{SS}}\|, \tag{12}$$

where z_{ω}^{SS} is the objective value of the stochastic problem with the second stage being the realization of scenario ω. In this sense, the first-stage decisions will be calculated $N{=}|\Omega|$ times, once per scenario in the original set.

III. Generalized Adaptive Partition-based Method

We propose to divide the full space of random values, either discrete or continuous, into partitions represented by P. Using similar jargon from the scenario reduction techniques, a representative scenario is defined by a partition P. They can be used for building the expected cost function (3) as

$$\mathbb{E}[\mathcal{Q}(x,\xi)] \approx \sum_{P} \mathbb{P}(P)\cdot \mathcal{Q}(x,\mathbb{E}[\xi|P]), \tag{13}$$

where the partition's second stage problem takes the form

$$\mathcal{Q}(x,\mathbb{E}[\xi|P]) = \min_{y\geq 0}.\{q^{\top}y|Wy = h^{P} - T^{P}x\}. \tag{14}$$

A fundamental question emerges: *how should we select the partitions P to have a good approximation (or exact value) of the original problem?*

We base our partition selection criteria (a.k.a. scenario reduction in other related works) on the theoretical finding from the work of Ramirez-Pico and Moreno [11]. The authors proved that for a two-stage optimization problem it is always possible to find a finite optimal partition that satisfies (13) *exactly*, i.e., with no approximation. The optimal partition set can be found when, for the optimal primal and dual solutions $\bar{x}$ and λ, the following conditions are satisfied for each element P:

$$\Big(\mathbb{E}\big[h^{\xi}|P\big]\Big)^{\top}\big(\mathbb{E}\big[\lambda^{\xi}|P\big]\big) = \mathbb{E}\Big[{h^{\xi}}^{\top}\lambda^{\xi}|P\Big] \tag{15a}$$

$$\bar{x}\Big(\mathbb{E}\big[T^{\xi}|P\big]^{\top}\mathbb{E}\big[\lambda^{\xi}|P\big]\Big) = \bar{x}^{\top}\mathbb{E}\Big[{T^{\xi}}^{\top}\lambda^{\xi}|P\Big] \tag{15b}$$

It is worth to mention that conditions (15) are always fulfilled in two cases:

1) If for each partition the value of the random parameters remains the same throughout the partition, i.e.,

$$h^{P}{=}h^{\xi}{=}\mathbb{E}\big[h^{\xi}\big] \text{ and } T^{P}{=}T^{\xi}{=}\mathbb{E}\big[T^{\xi}\big]. \tag{16a}$$

2) If the optimal dual value λ^{ξ} does not change within the partition, i.e.,

$$\lambda^{P}{=}\lambda^{\xi}{=}\mathbb{E}\big[\lambda^{\xi}\big]. \tag{16b}$$

Note that the first case leads to having a partition for each scenario, becoming impractical for realistic implementations. On the other hand, since the second case (16b) indicates that the optimal dual variable must not change within the partition, the scenarios can be classified by the values of their optimal dual variables, providing a way to discretely group them.

The discrete scenario partition is guaranteed in most power systems problems since the dual variables tend to take non-continuous values, e.g., under marginal pricing, the electricity price reflects the cost of the marginal generation unit, and this changes discretely with the load level.

Once the optimal partition set that satisfies the conditions (15) is found, problem (1) is equivalent to:

$$\min_{x\in\mathcal{X}}.\left\{c^{\top}x + \sum_{P}\mathbb{P}(P)\cdot\mathcal{Q}(x,\mathbb{E}[\xi|P])\right\} \tag{17}$$

To find a partition set defined as $\mathcal{P}$, that makes problem (17) exactly equivalent to problem (1), Ramirez–Pico and Moreno [11] proposed a sequential algorithm that iteratively updates the partition set $\mathcal{P}$. The algorithm, refereed by the authors as **generalized adaptive partition-based method** (GAPM), is summarized in Fig. 1 and described as follows:

Step 0) Initialization:

During the initial step, the iteration counter is initialized, $k{=}1$. The partition $\mathcal{P}^{(k)}$ is equal to the original uncertainty set Ω. Finally, the upper and lower bounds are respectively set to negative and positive infinity, $z_L^{(k=1)}{=}{-\infty}$, and $z_U^{(k=1)}{=}\infty$.

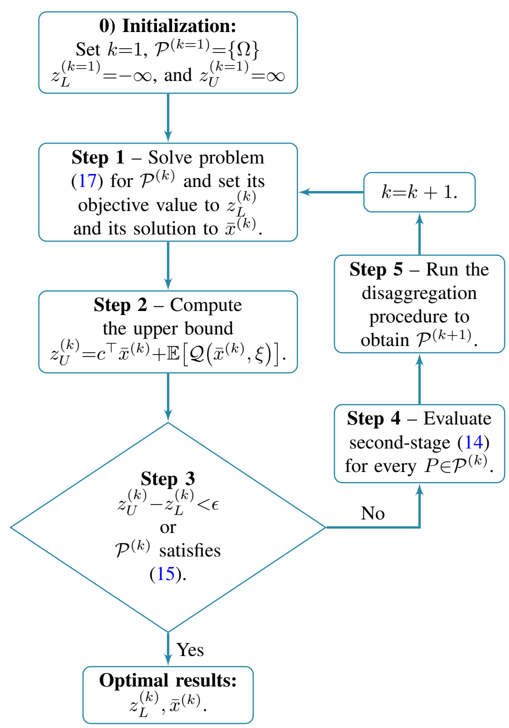

Figure 1. Flowchart of the generalized adaptive partition-based method

Step 1) Deterministic equivalent solution:

With the computed partition, solve the deterministic equivalent problem (17), setting its objective value as the problem's lower bound $z_L^{(k)}$ and storing the optimal optimal solution $\bar{x}^{(k)}$.

Step 2) Upper bound computation:

With the optimal solution of problem (17), $\bar{x}^{(k)}$, compute the problem's upper bound with the expression:

$$z_U^{(k)} = c^\top \bar{x}^{(k)} + \mathbb{E}\left[\mathcal{Q}\left(\bar{x}^{(k)}, \xi\right)\right] \quad (18)$$

Step 3) Convergence test:

If it is possible to compute the upper bound in **Step 2**, then verify whether the gap between the upper and lower bounds is below the convergence tolerance ϵ. However, if it is not possible to evaluate the upper bound on **Step 2**, the convergence to the optimal solution is confirmed by verifying that partition $\mathcal{P}^{(k)}$ satisfies conditions (15).

When one of the tests passes, the algorithm terminates, yielding $\bar{x}^{(k)}$ and $z_L^{(k)}$ as the ϵ–optimal solution and objective value. Otherwise, the algorithm advances to **Step 4**.

Step 4) Subproblems evaluation:

Solve the second-stage subproblem (14) for each partition $P \in \mathcal{P}^{(k)}$. The partitions' results are then evaluated in **Step 5** to determine the new disaggregated partitions.

Step 5) Disaggregation:

The disaggregation procedure can follow two different approaches depending on the problem structure:

(i) **Sensitivity-analysis partitioning (GAPM-SP):** by dividing the partitions $P \in \mathcal{P}^{(k)}$ based on a dual sensitivity analysis of the uncertain parameter h^P for subproblem (14). This analysis yields the bounds for h^P within which the dual solution does not change. Thus setting the limits that will subdivide each of the current partitions.

(ii) **Dual-value partitioning (GAPM-DP):** by identifying the possible values that the dual variable associated with the second stage subproblem (14) can obtain as a step function of the uncertain parameter h^P, and splitting the partition P based on this function's domain intervals.

After the disaggregation procedure, the step counter is updated, $k = k + 1$ and the algorithm proceeds to **Step 1**.

IV. Numerical Tests

To compare the performance of the scenario reduction techniques based on the forward-selection algorithm with the GAPM, we present a simple wind investment problem (19) analyzed for two different scenario sets of cardinality 15 and 10 000. Additionally, we also employ the popular K-means technique [12], and the representative scenario blocks, based on load- and wind-duration curves, described by Baringo and Conejo in [13].

The objective of the investment model (19) is to determine the optimal wind capacity x that will minimize both the investment costs, $x{\cdot}C^{\text{inv}}$, and the expected operational costs, $Q(x, \xi)$, under the uncertainty ξ. The uncertainty vector ξ is two-dimensional, capturing normalized wind production and load demand uncertainty, $\xi = [W, D]$. However, abusing notation, we add superscript ξ to random input variables for clarity. For instance W^ξ and D^ξ refers to the random wind and demand respectively.

The operational costs, (19b), can be expressed as the sum of the generation $(g{\cdot}C^{\text{g}})$, wind spillage $(w^{\text{sp}}{\cdot}C^{\text{sp}})$, and load shedding $\left(l^{\text{sh}}{\cdot}C^{\text{sh}}\right)$ costs. The load balance, (19c), ensures that the requested uncertain demand $\left(D^\xi\right)$ is matched by the sum of the uncertain wind generation $\left(x{\cdot}W^\xi\right)$ and the electric generation (g), wind spillage (w^{sp}) and load shedding $\left(l^{\text{sh}}\right)$. The electric generation is limited by the generation capacity $\left(\overline{G}\right)$ (19d). Whereas the wind spillage and load shedding are bounded by the uncertain wind generation and demand levels, respectively in (19e) and (19f).

$$z = \min_{x \geq 0} \left\{x{\cdot}C^{\text{inv}} + \mathbb{E}[\mathcal{Q}(x, \xi)]\right\} \quad (19a)$$

where

$$\mathcal{Q}(x, \xi) = \min_{g, w^{\text{sp}}, l^{\text{sh}}} \left\{g{\cdot}C^{\text{g}} + w^{\text{sp}}{\cdot}C^{\text{sp}} + l^{\text{sh}}{\cdot}C^{\text{sh}}\right\} \quad (19b)$$

$$\text{s.t.} \quad D^\xi = x{\cdot}W^\xi + g - w^{\text{sp}} + l^{\text{sh}} \quad (\lambda) \quad (19c)$$

$$0 \leq g \leq \overline{G} \quad (19d)$$

$$0 \leq w^{\text{sp}} \leq x{\cdot}W^\xi \quad (19e)$$

$$0 \leq l^{\text{sh}} \leq D^\xi \quad (19f)$$

The computational tests were performed in the language Julia 1.6.0., with the optimization package JuMP v0.21.6 and the solver Gurobi v9.1.1. The test machine features an Intel(R) Xeon(R) Gold 6148 CPU @ 2.40GHz, 2394 Mhz, 20 Core(s), 40 Logical Processor(s), Total Physical Memory 256 GB.

A. *Case 1: 15 scenarios*

Figure 2 presents the scenarios' partitioning by the K-means method and the SRTs based on the forward selection algorithm. The stars indicate the representative scenarios and centroids, while their sizes depict their associated probability. The numerical results of employing the scenario reduction and clustering techniques for the uncertainty set with 15 points is presented in Table I. Given the low number of scenarios, the GAPM with dual value partitioning (GAPM–DP) was employed. The processing and solution times are not reported due to being lower than 1 second. To present a fair comparison, the number of representatives scenarios and clusters for the SRTs and K-means methods is set to 5 since the GAPM–DP obtained an optimal partition set of cardinality 5. The number of chosen blocks was set to 6 to maintain an even division of the uncertainty space.

It is notable how, for this small problem with a reduced number of scenarios in the uncertainty space, the SRTs based on the forward selection algorithm result in the most significant gaps from both the optimal investment decision and objective value. The reason for this behavior is that the uncertainty is not only presented in the right-hand side parameters but also in a parameter multiplying the wind investment decision in the second-stage power balance. Thus, the SRTs based on the forward selection algorithm could be failing to recognize the scenarios' effect on the first-stage decision by either looking

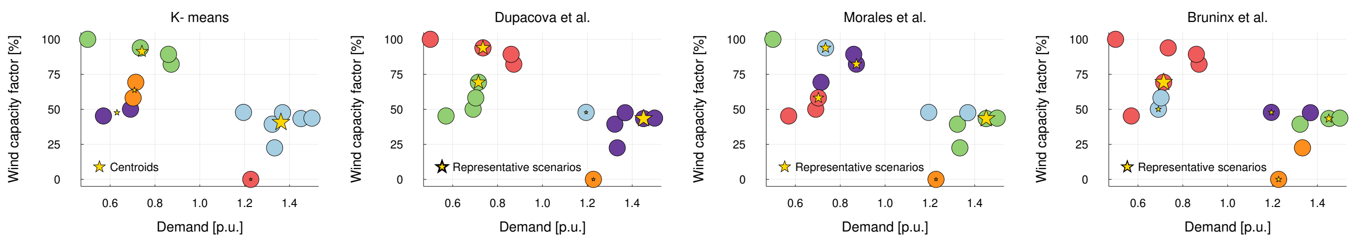

Figure 2. Partitions for different scenario reduction techniques

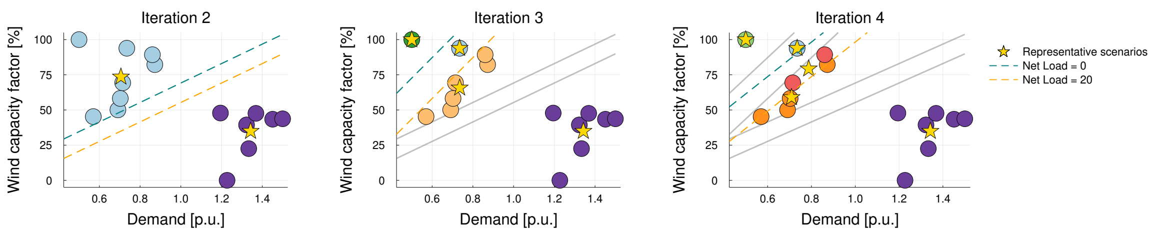

Figure 3. Iterative partitioning by the generalized adaptive partition method (GAPM)

only at the uncertainty data (Dupačová *et al.*), the aggregated deterministic-equivalent problem (Morales *et al.*), or the disjoint scenario analysis (Bruninx *et al.*). As seen in Fig. 2, each of the SRTs gives completely different representative scenarios based on which aspect of the stochastic optimization problem is considered. Interestingly enough, due to the data nature and the problem structure, the simpler scenarios' block division results in a lower optimality gap than the above mentioned SRTs. The K-means method, which divides the scenarios into clusters based on how they are grouped in the uncertain bi-dimensional space, results in lower optimality gaps than the SRTs, except the GAPM–DP, but only when considering the mean of the ten experiments for this approach. Due to its heuristic nature, the K–means clustering technique has been reported to yield widely different results every time it is ran, rendering it unreliable unless a large number of experiments are averaged [14].

The optimal reduced set obtained by the GAPM–DP is composed of 5 partitions and its evolution through its 4 iterations is shown in Fig. 3. The optimal GAPM-DP partitions

TABLE I
RESULTS SUMMARY FOR SCENARIO PARTITION METHODS BASED ON 15 DATA POINTS

Method	$\lvert\mathcal{P}^*\rvert$	x^* [kW]	Gap [%]	Objective [kEUR]	Gap [%]
All scenarios	15	78.26	–	699.13	–
Blocks [13]	6	77.69	-0.73	673.60	3.65
K-means[a]	5	79.75	1.90	665.75	-0.59
Dupačová *et al.* [6]	5	74.23	-5.15	674.69	3.50
Morales *et al.* [9]	5	81.73	4.43	654.99	-6.31
Bruninx *et al.* [10]	5	74.23	-5.15	686.52	-1.80
GAPM–DP	5	78.26	0.00	699.13	0.00

[a] 10 experiments were run. We present the results' mean value.

are obtained by creating hyperplane cuts in the bi-dimensional uncertainty space based on the value of the dual variables for the scenarios. For the wind investment problem (19), let the system's net load $N^\xi{=}D^\xi - x{\cdot}W^\xi$, then the possible values of the dual variable λ can be classified as:

$$\lambda = \begin{cases} -C^{\text{sp}}, & \text{if } N^\xi < 0, \\ C^{\text{g}}, & \text{if } 0 \leq N^\xi < \overline{G}, \\ C^{\text{sh}}, & \text{if } \overline{G} \leq N^\xi. \end{cases} \tag{20}$$

In each iteration k of the GAPM-DP once the value of the wind investment has been obtained for the representative partition $\mathcal{P}^{(k)}$, the value of the net load N^ξ can be used to define the hyperplanes that will divide the bi-dimensional uncertainty space and the scenarios that fall under the same subspace, i.e., the scenarios with the same value for the dual variable λ will be assigned to the same partition. The iterative partitioning of the uncertainty space based on the values of the dual variable λ can be clearly seen in Fig. 3, where $\overline{G}{=}20$. On each iteration the first-stage decision x (wind capacity) is updated, yielding new cuts based on the updated net load regions. Note that newly introduced cuts are combined with the existing ones to create a more detailed sectioning of the uncertainty space.

B. Case 2: 10 000 scenarios

The investment problem (19) is solved with different scenario reduction approaches assuming that vector ξ follows a bivariate normal distribution from which a random sample of 10 000 scenarios is taken; the results are presented in Table II. As in the case with 15 scenarios, the SRTs presented by Dupačová *et al.* and Morales *et al.* present gaps on the first-stage variable larger than 5%. The SRT proposed by Bruninx *et al.* recovers the optimal value for the first-stage variable with negligible gap (0.053%). Unfortunately, the obtained accuracy from the Bruninx SRT cannot be guaranteed for all applications and uncertainty sets, as seen in the previous test case with just 15 scenarios in the original set. Additionally, even though the STRs based on the forward selection algorithm could be efficiently solved once the representative scenarios are obtained, the (overhead) time necessary to obtain these scenarios is considerably larger than the time employed by the other analyzed methods. The larger scenario processing time corresponds to the the forward selection algorithm's complexity increases quadratically with both the number of representative scenarios and scenarios in the original uncertainty set [8].

The use of load- and wind-duration-based scenario blocks and the K–means method remain close to the optimal solutions with an error comparable to those of the sample average approximation (SAA) tests performed with larger sample bases of 100 and 1 000 scenarios while providing considerably faster solutions.

The GAPM with partitioning based on sensitivity analysis and the GAPM based on dual-value comparison converge to an optimal partition of cardinality several orders of magnitude smaller than the original sampling set. However, the GAPM–DP finds an optimal reduced set three times smaller than that found by the GAPM–SP. This corresponds to the fact that in the sensitivity analysis of the GAPM–SP, the cuts are performed within each partition and not in a transverse manner that influences several existing partitions at the same time. Thus, the cuts derived from the sensitivity analysis cannot recover information from scenarios in different partitions.

V. Conclusions

The presented generalized adaptive partition-based method (GAPM) allows for the optimal partitioning of an uncertainty set in two-stage stochastic optimization problems with fixed recourse. In order to find the optimal partition, two easily verifiable conditions must be met. Even though this is an iterative scenario reduction approach, when compared with techniques based on the forward selection algorithm and their scenario processing time, this method converges quickly to a representative partition with an optimality certificate. However, depending on the problem structure, such partitions can be obtained in multiple ways.

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TABLE II
Results summary for partition methods based on a test sample of 10 000 scenarios

Method	$\lvert\mathcal{P}^*\rvert$	x^* [kW]	Gap [%]	Objective [kEUR]	Gap [%]	Solution [s]	Overhead [s]	Out-of-sample cost [kEUR]
SAA–10 000	10 000	96.66	–	669.676	–	1.255	–	670.596
SAA–1 000[b]	1 000	98.301 [94.703, 101.392]	1.698	667.537 [657.78, 695.634]	0.319	47.614	–	669.189 [669.008, 669.638]
SAA–100[b]	100	99.093 [90.358, 108.864]	2.517	659.377 [609.974, 720.689]	1.538	40.375	–	670.209 [669.02, 673.268]
Scenario blocks [13]	18	94.012	-2.740	662.739	1.036	0.012	1.711	670.732
K-means[b]	18	97.920 [92.275, 104.051]	1.304	665.747 [664.978, 666.876]	0.587	0.014	0.219	670.909 [670.521, 672.077]
Dupačová *et al.* [6]	17	103.458	7.033	670.796	0.167	0.016	160.040	669.748
Morales *et al.* [9]	17	110.149	13.955	725.498	8.336	0.015	161.543	673.528
Bruninx *et al.* [10]	17	96.609	0.053	669.672	1.369	0.022	160.008	668.531
GAPM – sensitivity partition	49	96.609	0.053	669.672	0.001	2.812	–	670.518
GAPM – dual value partition	17	96.680	0.021	669.672	0.001	2.211	–	670.517

[b] 10 experiments were run. We present the results' mean value and their range.