Optimal Power Flow with Substation Reconfiguration

Anton Hinneck, Basel Morsy, David Pozo, Janusz Bialek
Center for Energy Science and Technology
Skolkovo Institute of Science and Technology
Moscow, Russia
anton.hinneck@skoltech.ru

***Abstract*—Substation reconfiguration may reduce congestion and therefore reduce the cost of economic dispatch. In this paper we propose a novel method that can be applied to any substation configuration using a mixed-integer linear model. Our methodology generalizes substation reconfiguration to include both more traditional transmission switching and bus splitting. The problem is NP-hard but we show that using state-of-the-art tools we can reduce the computation time significantly. The method has been evaluated on several test cases, the IEEE 118-bus case being the largest. Optimal solutions were found within much smaller time frames than have been previously reported.**

***Index Terms*—Optimal bus splitting problem (OBS), Optimal Transmission Switching Problem (OTS), Mixed-Integer Programming (MIP), Topology control**

Nomenclature

Dual variables

$\overline{\alpha}_e$ Dual value of line e's maximal capacity.
$\underline{\alpha}_e$ Dual value of line e's minimal capacity.

Parameters

$\overline{\theta}_v$ Maximal voltage angle at bus v.
$\overline{F}_e$ Maximal transmission capacity of line e.
$\overline{P}_g$ Maximal generation capacity of generator g.
$\underline{\theta}_v$ Minimal voltage angle at bus v.
$\underline{P}_g$ Minimal generation capacity of generator g.
B_e Susceptance of line e.
c_g Generation costs of generator g.
d_v Load at bus v.
v_e^s Starting bus of line $e \in \mathcal{E} \cup \bar{\mathcal{E}}$.
v_e^t Terminal bus of line $e \in \mathcal{E} \cup \bar{\mathcal{E}}$.

Sets

$\bar{\mathcal{E}}$ Auxiliary transmission lines.
$\bar{\mathcal{W}}$ Auxiliary buses without busbars.
$\bar{\mathcal{W}}_v$ Auxiliary buses in SR of bus v without busbars.
$\mathcal{E}$ Original Transmission lines.
$\mathcal{E}_v^s$ Outgoing transmission lines at bus v.
$\mathcal{E}_v^t$ Incoming transmission lines at bus v.
$\mathcal{E}_v$ Transmission lines connected to bus v.
$\mathcal{G}$ Generators.
$\mathcal{V}$ Original Buses.
$\mathcal{W}$ Auxiliary buses.
$\mathcal{W}_v^{\mathcal{B}}$ Auxiliary busbars in SR of bus v.
$\mathcal{W}_v^{\mathcal{E}}$ Auxiliary feeder buses in SR of bus v.
$\mathcal{W}_v^{\mathcal{G}}$ Auxiliary generator buses in SR of bus v.
$\mathcal{W}_v^{d}$ Auxiliary load buses in SR of bus v.
$\mathcal{W}_v$ Auxiliary buses in SR of bus v.

Variables

θ_v Phase angle at bus v.
f_e Power flow on transmission line e.
p_g Power generation of generator g.
x_g Operational status of line e.

This work was partially funded by the Skolkovo Institute of Science and Technology as a part of the Skoltech NGP Program (Skoltech-MIT joint project).

978-1-6654-3597-0/21/$31.00 ©2021 IEEE

I. Introduction

Solving optimal power flow (OPF) models is one of the most fundamental steady-state power system operation performed by utilities. These are typically minimizing a cost function that is subject to a certain set of constraints. Previous work has shown, that changing a network's topology can have desirable effects on a system's state, by redirecting power flows to remove congestion, reducing voltages, or improving operational security [1], [2], [3]. Main challenge when employing switching in actual systems, is that discrete changes to a system's topology cause non-continuous changes in the system's operating point, which can cause instability. Concerns regarding the reliability of systems are raised [4]. Fisher et. al. [1] proposed a mixed-integer programming model to decrease congestion in transmission systems by solving the *optimal transmissions switching* (OTS) problem. Binary variables are used to model the operational status of transmission lines (operational/non-operational). This model was extended by accounting for multiple periods, investments, security-constrained unit commitment, and uncertainty [5], [6], [7]. Problems containing integer (binary) variables can be difficult to solve as they are non-convex. Switching problems particularly were shown to be NP-hard [8].

The *optimal bus splitting* (OBS) problem is another switching problem that describes how an original bus in a substation can be split into one or more sections [9], each maintaining connections to different lines. Switching operations in substations provide independent control over a variety of components in a transmission grid (e.g., generators, loads, and transmission lines) and were employed to relieve voltage violations and overloads [10]. In [10], the authors model bus splitting such that an original bus is replaced by new buses for every connected line, load and shunt element. Small impedance lines are introduced to connect the newly generated buses, dividing these elements into two groups. This methodology was extended using a substation model in [11], [12], which controls the assignment of generators, lines, and loads to individual busbars. Following their methodology, an original bus is replaced by a substation representation with two busbars and the respective elements. These are connected by switchable zero-impedance lines (ZILs). A binary variable denotes to which of the two busbars an element (e.g. generator) is connected. Thus, more or less busbars cannot be considered. To our best knowledge, mathematical formulations able to model arbitrary real-world substation designs have not been proposed. In the literature on this topic, not more than two busbars are considered in optimization models. Hence, a bus can at most be split into 2 new buses. High-voltage substations that have to meet high standards with regards to operational security can have 4 or even 5 busbars [13].

Paper Contributions: We propose a generalized bus splitting methodology that can accommodate any real-world substation configuration (also referred to as OBS). Based on our approach, we show that OTS is a special case of OBS for 1-busbar substations and provide insights by conducting several case studies. Our generalization comes at the cost of computational complexity however. More binary variables must be introduced modelling a certain system than using any other approach. We show that OTS and OBS are closely related, which can be exploited to accelerate the solution process.

II. Preliminary Optimal Power Flow Models

The foundation of the model developed in this paper is the direct current optimal power flow approximation (DCOPF) presented as Model 1. In this model, equation (1a) denotes the objective, which is minimizing costs of power generation. Equation (1b) denotes the Kirchhoff's current law and equation

Model 1. DCOPF	[LP]

$$z_1^* = \min \sum_{\mathcal{G}} c_g(p_g), \quad \text{s.t.:} \tag{1a}$$

$$\sum_{\mathcal{E}_v^{\mathrm{A,s}}} f_e - \sum_{\mathcal{E}_v^{A,t}} f_e = \sum_{\mathcal{G}_v} p_g - d_v, \quad \forall\ v \in \mathcal{V} \tag{1b}$$

$$B_e(\theta_{v_e^{\mathrm{s}}} - \theta_{v_e^{\mathrm{t}}}) = f_e, \quad \forall\ e \in \mathcal{E} \tag{1c}$$

$$-\overline{F}_e \leq f_e, \quad \forall\ e \in \mathcal{E} \quad [\underline{\alpha}] \tag{1d}$$

$$f_e \leq \overline{F}_e, \quad \forall\ e \in \mathcal{E} \quad [\overline{\alpha}] \tag{1e}$$

$$\underline{\theta}_v \leq \theta_v \leq \overline{\theta}_v, \quad \forall\ v \in \mathcal{V} \tag{1f}$$

$$\underline{P}_g \leq p_g \leq \overline{P}_g, \quad \forall\ g \in \mathcal{G} \tag{1g}$$

(1c) the Kirchhoff's voltage law. The power flow on transmission lines and the production by generators are constrained by (1d), (1e) and (1g) respectively. To incorporate the operational status of transmission lines, this basic model was extended by [1]. Equations (1c), (1d) and (1d) are replaced by constraints (2c), (2d) and (2e), as shown in Model 2.

Model 2. OTS (Fisher et. al. [1])	[MILP]

$$z_2^* = \min \sum_{\mathcal{G}} c_g(p_g), \quad \text{s.t.:} \tag{2a}$$

$$\text{(1b)},\ \text{(1f)}\ \&\ \text{(1g)} \tag{2b}$$

$$B_e(\theta_{v_e^{\mathrm{s}}} - \theta_{v_e^{\mathrm{t}}}) + (1 - x_e)M_e \geq f_e, \quad \forall\ e \in \mathcal{E} \tag{2c}$$

$$B_e(\theta_{v_e^{\mathrm{s}}} - \theta_{v_e^{\mathrm{t}}}) \leq f_e + (1 - x_e)M_e, \quad \forall\ e \in \mathcal{E} \tag{2d}$$

$$-\overline{F}_e x_e \leq f_e \leq \overline{F}_e x_e, \quad \forall\ e \in \mathcal{E} \tag{2e}$$

$$x \in \{0, 1\}, \quad \forall\ e \in \mathcal{E} \tag{2f}$$

Additionally, integer constraint (2f) is added. A variable x_e denotes the operational status of a line e. Line e is operational if $x_e = 1$ and non-operational if $x_e = 0$. Constraints (2c) and (2d) only apply, if $x_e = 1$. If $x_e = 0$, (2c) and (2d) are non-binding due to a sufficiently large M_e.

III. Optimal Bus Splitting Modeling Framework

A transmission network can be represented by the graph $\mathcal{N}(\mathcal{E}, \mathcal{V})$, where $\mathcal{E}$ denotes the set of edges, also referred to as (transmission) lines, and $\mathcal{V}$ denotes the set of vertices, also referred to as buses. The model proposed in this paper requires a prior operation on the initial transmission network to derive a so-called augmented network model (ANM). Solving the proposed model on this ANM yields an optimal topology - considering substation designs. Having obtained a solution, reducing the resulting graph stores the information eliminating redundancies. Additionally, DCOPF can then be solved on the reduced graph to derive dual values. The method proposed, given an arbitrary power grid, consists of three major steps to consider full substation reconfiguration:

1) Augmenting the network model,
2) Solving the OBS to optimize switching states and
3) Reducing the network model.

In the following, each component of the procedure will be described individually, starting with expanding an arbitrary initial network to an ANM.

A. Augmenting the network model

Given the network of a power system, we want to be able generate a graph representing a substation, which will be referred to as substation representation (SR), for an arbitrary bus v. By performing switching operations in SRs, we want to account for all possible reconfigurations that would be possible considering an arbitrary substation's design. This requires, that the SRs are constructed such that arbitrary real-world substation designs can be accommodated. Furthermore, they must be constructed systematically, so that an arbitrary network's topology can be optimized.

1) A general substation representation: Any bus $v \in \mathcal{V}$ can have the following components/properties assigned to it; a load d_v, a set of incoming and outgoing transmission lines $\mathcal{E}_v$ and a set of generators $\mathcal{G}_v$. Information on these components and properties can typically be obtained from an original data set. To generate a substation representation (SR), the set of busbars (or busbar segments) $\mathcal{B}_v$, with $|\mathcal{B}_v|$ denoting the number of elements, is also required. This is not widely available. In test cases (if not stated otherwise), we chose 2 busbars for the experiments in this paper. If information would be available, any number of busbars could be set when constructing a SR for a specific bus v, as long as $|\mathcal{B}_v| \geq 1$. As in [14], we generate a new (auxiliary) bus $w_v \in \mathcal{W}_v$ for each of those aforementioned properties or elements. We omit small impedances however and connect these buses using auxiliary zero-impedance lines (ZILs), which are switchable. The auxiliary buses in a SR are assigned to one of the following sets: $\mathcal{W}_v^{\mathcal{B}}$, $\mathcal{W}_v^{\mathcal{E}}$, $\mathcal{W}_v^{\mathcal{G}}$, $\mathcal{W}_v^d$ and $\bar{\mathcal{E}}_v$, which will be discussed in the following.

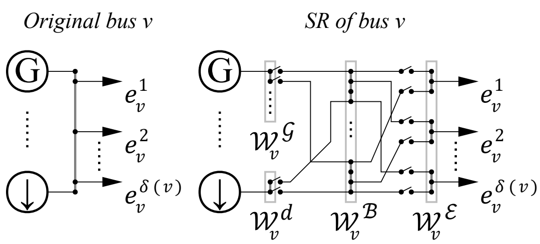


Figure 1. Sets of auxiliary buses

Figure 1 displays the auxiliary buses and lines. It can also be seen how auxiliary buses are constructed and assigned to sets - which is required for modelling. An auxiliary bus w is created for every busbar. These newly created buses are contained in $\mathcal{W}_v^{\mathcal{B}}$. The same applies for for every line that is connected to original bus v. Parameter $\delta(v)$ denotes the degree of bus v. It is equal to the number of auxiliary buses in $\mathcal{W}_v^{\mathcal{E}}$. All auxiliary buses created for each distinct generator $g \in \mathcal{G}_v$ are contained in $\mathcal{W}_v^{\mathcal{G}}$, whereas the auxiliary bus for load d_v is contained in $\mathcal{W}_v^d$. The set of all auxiliary buses is denoted by $\mathcal{W}$, with $\mathcal{W} = \mathcal{W}_1 \cup \cdots \cup W_{|\mathcal{V}|}$, whereas the set of all auxiliary lines is denoted by $\bar{\mathcal{E}}$. Define $\bar{\mathcal{W}}_v = \mathcal{W}_v^{\mathcal{E}} \cup \mathcal{W}_v^d \cup \mathcal{W}_v^{\mathcal{G}}$ and $\bar{\mathcal{W}} = \bar{\mathcal{W}}_1 \cup \bar{\mathcal{W}}_2 \ldots \bar{\mathcal{W}}_{|\mathcal{V}|}$. When constructing the SR of bus v, auxiliary buses in $\mathcal{W}^{\mathcal{B}_v}$ are created first. Whenever an auxiliary bus $w \in \bar{\mathcal{W}}_v$ is created while constructing the SR of bus v, auxiliary lines are created connecting w to each $w' \in \mathcal{W}_v^{\mathcal{B}}$. Hence, we create $(|\mathcal{W}_v^{\mathcal{E}}|+|\mathcal{W}_v^{\mathcal{G}}|+|\mathcal{W}_v^d|)*|\mathcal{W}_v^{\mathcal{B}}|$ ZILs for an individual substation representation, which requires an equal number of binary variables. The resulting problem can be very complex. Concerns regarding the computational tractability of this approach are addressed in section IV. Using this augmented network model, arbitrary technical aspects of substations can be considered. Some substations are merely used as switching stations [15]. Substations connecting equal components can differ by the amount of circuit breakers and busbars. The layout of a substation influences its costs, reliability, maintainability and security [16]. Even though most designs are derived from one of a few fundamental types (breaker-and-a-half; ring bus; double bar, single breaker; single bus, double breaker), substation designs are quite diverse. A generalized modelling method must be able to represent any fundamental case and variation thereof. This is achieved by the construction outlined above.

B. A mixed-integer formulation

The MIP presented in this section is solved after an ANM was constructed, which means that selected buses were transformed into a substation representation. As new lines and buses are constructed, introducing necessary parameters for a new line e must be addressed. Susceptance B_e and capacity limit $\overline{F}_e$ must be chosen appropriately. $\overline{F}_e$ can be set to a large enough value such that the power flow within a substation is not constrained. Regarding the susceptance, two approaches were already applied in previous works for B_e. An additional line was assumed to have a small impedance [14] or to be 0 [11]. In our experiments both approaches showed to be feasible. Assuming 0-impedance has advantages. First, small parameters can lead to numerical instabilities. Secondly, adding new lines with impedance causes numerical differences when applying our method versus solving the fundamental models. Even though minor, this becomes an issue when solutions are to be exchanged between models.

Model 3. OBS	[MILP]

$$z_3^* = \min \sum_{G} c_g(p_g), \quad \text{s.t.:} \tag{3a}$$

$$\sum_{E_v^{A,s}} f_e - \sum_{E_v^{A,t}} f'_e = \sum_{G_v} p_g - d_v, \quad \forall\, v \in \mathcal{W} \tag{3b}$$

$$\text{(1f) \& (1g)} \tag{3c}$$

$$B_e(\theta_{v_e^s} - \theta_{v_e^t}) = f_e, \quad \forall\, e \in \mathcal{E} \tag{3d}$$

$$\theta_{v_e^s} \leq (1 - x_e)M_e + \theta_{v_e^t}, \quad \forall\, e \in \bar{\mathcal{E}} \tag{3e}$$

$$\theta_{v_e^s} + (1 - x_e)M_e \geq \theta_{v_e^t}, \quad \forall\, e \in \bar{\mathcal{E}} \tag{3f}$$

$$-\overline{F}_e x_e \leq f_e \leq \overline{F}_e x_e, \quad \forall\, e \in \bar{\mathcal{E}} \tag{3g}$$

$$-\overline{F}_e \leq f_e \leq \overline{F}_e, \quad \forall\, e \in \mathcal{E} \tag{3h}$$

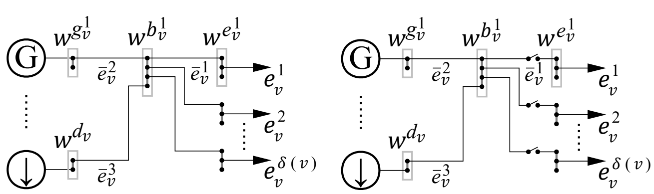
$$x_e \in \{0, 1\}, \quad \forall\, e \in \bar{\mathcal{E}} \tag{3i}$$

$$\{x_e : e \in \bar{\mathcal{E}} \wedge \boldsymbol{q}(e, w)\} \in \text{SOS1}, \quad \forall\, w \in \bar{\mathcal{W}} \tag{3j}$$

The required constraints for ZILs can be inferred from constraints (1d) and (1e) in Model 2. The full OBS formulation is shown in Model 3. For non-switchable lines (3d) applies, like in Model 1. Compared to Model 2, Model 3 requires adjustments. Constraints (3e) and (3f) are used to model the switching of ZILs. The phase angles for all buses are required to be equal if an active ZIL connects two auxiliary buses. Due to the modeling symmetry at the substation level, degenerated solutions (e.g., different splitting solutions with equal optimal cost) are expected, burdening the solution process's tractability. This can be prevented if any $w \in \bar{\mathcal{W}}_v$ can be at most connected to one busbar. This additionally excludes, otherwise, possible solutions that do not have to be explored. Consider elements in $\mathcal{W}_v^{\mathcal{E}}$. Given the SR for a bus v, any element in $\mathcal{W}_v^{\mathcal{E}}$ can at most be connected to one busbar. This can be be expressed as a special ordered set of type 1 (SOS1). Defining $\boldsymbol{q}(e, w) = ((v_e^s = w) \vee (v_e^t = w))$, yields constraints (3j). Condition $\boldsymbol{q}(e, w)$ can be verbalized as follows: "Line e starts or ends at bus w." We need to iterate over all lines that start or terminate at a specific auxiliary bus. Constraints (3b) must now apply for auxiliary buses. Note that constraints (3d) and (3h) are equal to Model 1 whereas constraints (3e) - (3g) & (3i) apply in Model 2 (in adjusted form). This is due to the different types of lines that must be considered. Switching can only be performed on ZILs $e \in \bar{\mathcal{E}}$. Lines $e \in \mathcal{E}$ must be considered with their original properties. This ensures that for a given topology the results of Model 1 - 3 are equal.

C. A generalization of DCOPF and OTS

Model 3 is a generalization of Model 1 and 2. Thus, Model 3 provides at least as much topology network flexibility than 1 or 2. In this section we show how this can be verified and discuss implications. Figure 2 shows the adjustments necessary in every SR (Figure 1) in a ANM so that Model 3 is equivalent to DCOPF (Model 1) or OTS (Model 2) respectively. Hereinafter, an original bus or line refers to an element that was present before the SR was constructed.



(a) DCOPF (b) OTS [1]

Figure 2. Representation of DCOPF and OTS

We briefly outline in the following under which conditions the Model 3 is equal to Model 1 and Model 2 respectively.

1) DCOPF: Model 3 is equal to Model 1 if the following is applied to a respective ANM and Model 3: First, a single busbar $w \in \mathcal{W}_v^{\mathcal{B}}$ must be selected. All auxiliary buses must be connected to w. This can be formalized as $x_e = 1 \leftrightarrow (\boldsymbol{q}(e, w) \wedge e \in \bar{\mathcal{E}}_v)$. As shown in Figure 2a, all auxiliary buses are then connected to a single busbar by non-switchable lines.

2) OTSP: Similar conditions are necessary to reproduce Model 2. In this case, given a single busbar $w \in \mathcal{W}_v^{\mathcal{B}}$ ($w^{b_v^1}$ in Figure 2 (right)), it must hold that $x_e = 1 \leftrightarrow (\boldsymbol{q}(e, w) \wedge e \in \bar{\mathcal{E}}_v \wedge (v_e^s \in \mathcal{W}_v^{\mathcal{G}} \vee v_e^t \in \mathcal{W}_v^{d}))$. This reads as follows: "Auxiliary lines are determined to be operational, if and only if they connect generators or loads to busbar w. Auxiliary lines connecting w and in- and outgoing feeders remain switchable (Figure 2b).

Remark. *Since any solution of Model 1 can be reconstructed using Model 2 and any solution of Model 2 can be obtained using Model 3, we observe that* $z_3^* \leq z_2^* \leq z_1^*$.

3) Generating solutions: Specifically, relationship III-C2 can be used to construct solutions for Model 3 given solutions of Model 2. If multiple busbars exist, a busbar $w \in \mathcal{W}_v^{\mathcal{B}}$ is chosen (e.g., lowest index). Then, operational statuses of respective ZILs must be set as described and visualized in Figure 2. As Model 2 is computationally less costly, we show in section IV that it can be used to obtain good-quality solutions for OBSP much faster.

D. ANM reduction

Once the switching statuses have been obtained using Model 3, the networks structure can be reduced. This step has two advantages: First, information is compressed. Second, when solving Model 1 on the reduced network, dual values $\bar{\alpha}$ and $\underline{\alpha}$ can be computed efficiently as it is an LP. During the reduction step, switching statuses are recovered. For every original bus v disconnected components are identified in the SRs. Here, a variation of depth-first-search (DFS) is used, which has a linear run time of $O(|\mathcal{V}| + |\mathcal{E}|)$. DFS is restarted after every node in a graph's disconnected component has been visited, until all nodes in the SRs' graphs have been visited. The result of this operation for the IEEE 5-bus case is shown in Figure 3.

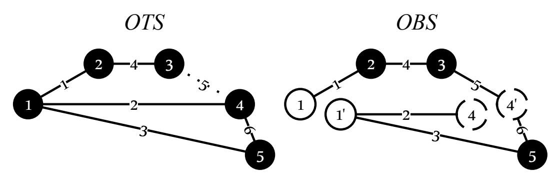


Figure 3. Comparison of OTS and OBS solutions

It can be seen clearly, that our methodology not only allows for switching lines but splitting buses.

IV. Numerical Analysis

Our method has been applied to the small IEEE 5-bus case for illustrative purposes, and then validated on larger networks. Our computer had 16 GB of RAM and an i7-7820X (hyper-threading enabled), clocked at 4.0 GHz. The software used was Gurobi 9.0.1, Julia 1.4.1, JuMP.jl 0.21.2, and Gurobi.jl 0.8.0. Experiments were performed on multiple test cases. For every test case, results of Model 1 - 3 were compared. Any feasible solution within 0.01% of a problem's lower bound is classified optimal. Before the results for all test cases are presented, the IEEE 5-bus case is described in detail. In the following, all SRs were constructed with 2 busbars.

A. The IEEE 5-bus case

This test case consists of 5 buses, 6 lines, and 5 generators. As a baseline, DCOPF and OTS were solved. The objective values z^* for all three models are summarized in Table I, first row. This table also summarizes computation times (ct). We provide a gap whenever an optimal solution is not obtained within 600s. It shows how close an incumbent solution is compared to a problem's lower bound. The number of binary variables is provided as a measure of complexity, whereas

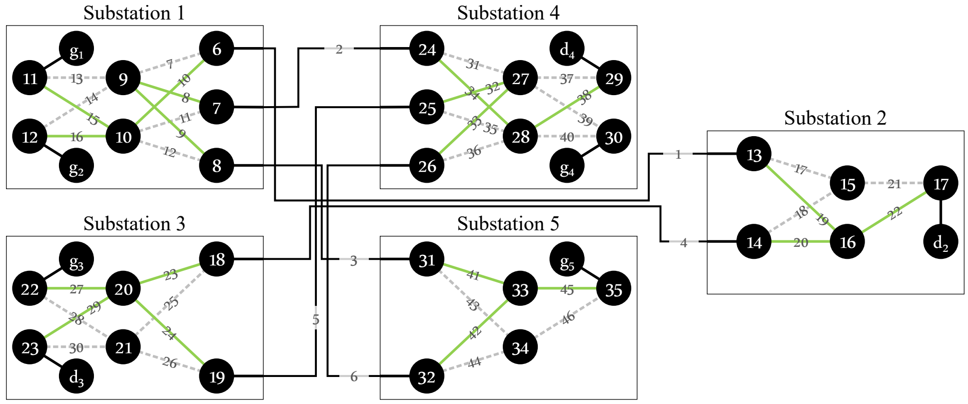


Figure 4. IEEE 5-bus case. Solution of the substations' reconfiguration.

TABLE I. Comparing results of selected test cases showing the number of switchable lines

		DCOPF	OTS			OBS		
Name	γ	z_1^* $[\frac{\$}{h}]$	z_2^* $[\frac{\$}{h}]$	# bin. vars.	ct [s]	z_3^* $[\frac{\$}{h}]$	# bin. vars.	ct [s] (gap [%])	$100(1 - z_3^* \backslash z_2^*)$
5_ieee	1.0	17479	14991	6	0.12	14810	40	0.06	1.2
14_ieee	0.55	2733	2558	20	0.12	2051	112	0.04	19.8
24_ieee_rts	0.5	57872	46087	38	0.18	44677	252	1.72	3.1
30_as	0.6	558	528	41	0.4	506	218	22.21	9.3
30_ieee	0.9	8065	7252	41	0.3	6412	218	0.37	11.6
57_ieee	0.3	38394	38161	80	0.99	38050	418	8.13	0.3
73_ieee	0.48	165550	135872	120	1.86	128866	780	600.0 (0.13)	5.2
118_ieee	0.74	96607	93139	186	67.07	93030	1050	490.37	0.1

the table's last column depicts the improvements (in $[\%]$) OBS achieves over OTS. To solve Model 3, the procedure outlined in this paper was applied. First, we expand the original network model to derive an ANM. A visualization of the generated graph is displayed in Figure 4. Solving Model 3 yields optimal operational statuses for all ZILs. Active ZILs are marked green. Reducing the network, as outlined in section III-D yields Figure 3 (right). In substation 1, two disconnected components can be identified. The vertices and edges of these components are $\mathcal{V}_1 = \{6, 10, 11, 12\}$ and $\mathcal{E}_1 = \{10, 15, 16\}$ as well as $\mathcal{V}_{1'} = \{8, 9, 7\}$ and $\mathcal{E}_1 = \{8, 9\}$. Comparing substation 1 in Figure 4 to buses 1 and 1' in Figure 3, illustrates how much redundant information can be compressed for the 5-bus network. Having derived the reduced network model, DCOPF can be solved to provide dual information.

In this specific case, the OBS is able to remove all congestion in the system. This cannot be achieved by transmission switching alone. Improvements produced by a more general model might be possible if at least one dual value in $\overline{\alpha}$ or $\underline{\alpha}$ is greater than 0. Reducing the ANM, dual information is available for the OBS solution. A similar technique is applied after solving Model 2. Here, we solve Model 1, only including active lines. The dual values for DCOPF are $\overline{\alpha}_6 = 6232.2$ and for the OTS $\underline{\alpha}_1 = 1500$. For the OBS solution, the dual values are equal to 0. These results confirm that congestion regarding transmission lines was removed completely utilizing our approach.

As indicated by the previously stated dual values, the only congested line when solving the DCOPF is line 6. This can be confirmed in Table II. Line 6 is the only line used at full capacity. For the OTS, the same applies. The capacity limit of lines 1 shows a dual value greater than 0. We can observe that this line's capacity is fully utilized as well. Having solved OTS on the system, the dual values of all line capacity constraints are 0. Moreover, no line is utilized fully. This means, that the objective cannot be improved by further topological optimization. Note, that the direction of the power flows might change, dependent on the topology. This can be observed for line 4 in the OTS solution. This last statement is confirmed by the results displayed in Table III. This table summarizes information on the individual generators. It shows that all required power is produced by the generators with lowest variable costs. This is not the case after solving the DCOPF or OTS: Generator 5 cannot be utilized fully.

TABLE II. Line utilization $[\%]$ for the IEEE 5-bus case

Line e	DCOPF	OTS	OBS
1	64.4	100.0	52.5
2	44.46	37.4	93.9
3	55.63	84.37	93.9
4	9.86	23.47	21.13
5	8.08	0.0	46.95
6	100.0	98.0	81.63
Objective value $[\frac{\$}{h}]$	*17479*	*14991*	*14810*

The 5-bus case is comparatively small. A large concern are cases with more transmission lines, as every transmission line (OTS) or ZIL (OBS) corresponds to a binary variable.

TABLE III. Generation p $[p.u.]$ for the IEEE 5-bus case

g	c_g	$\overline{P}_g$	DCOPF	OTS	OBS
1	14.0	0.4	0.4	0.4	0.4
2	15.0	1.7	1.7	1.662	1.7
3	30.0	5.2	3.235	2.0	1.9
4	40.0	2.0	0.0	0.0	0.0
5	10.0	6.0	4.665	5.938	6.0

B. Tests on larger networks

Due to the computational complexity of the problem, evaluating the method on an array of larger problems is required to reason its feasibility. Improvements can only be expected if a system is congested. In particular, any of the constraints (1d) and (1e) must be binding. We utilized parameter γ, with $0 \leq \gamma \leq 1$, to stress the systems. It is used to scale down lines' capacity ratings. Parameter γ used in the numerical experiments is reported in Table I for every test case. First of all, earlier findings are confirmed. Our method improves on the OTS in all cases. Here must be mentioned that we ensured using γ that the systems are congested sufficiently. The OTS, on the other hand, improves on the DCOPF solution in every instance. Table I shows the complexity of OBS when compared to OTS - the number of binary variables (or switchable lines). Our methodology and model leads to higher binary variable counts. Due to the SR, 40 instead of 6 lines are required for the 5-bus case. As the problem instances get larger, this difference increases as well. However, as the computation times show, obtaining optimal OTS solutions can take longer. Providing good warm-start solutions helped the process. The largest case considered is the 118-bus case. The resolution process for Model 3 on that problem instance is displayed in Figure 5.

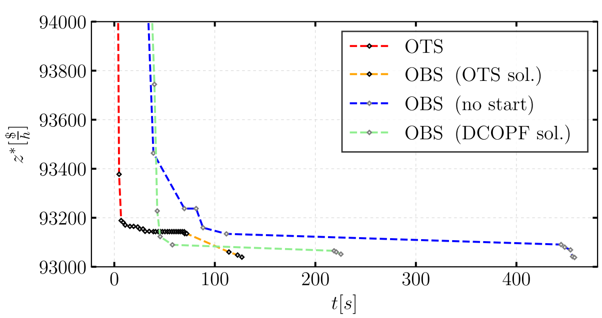


Figure 5. Solution search evolution of the 118-bus case

As a baseline, we solved the OBS without any initial solutions (blue). Constructing a mipstart as outlined in section III-C using the DCOPF topology (all lines active) improves the process (green). As described, any OTS problem solution can be converted into a OBS problem solution as well. We obtained better results solving OTS problem (red) until optimality to then construct a solution for Model 3 and continue solving OBS (orange) until reaching an optimal solution. Here, constructing good quality solutions using computationally less expensive models reduces the computation times by more than 70%. We can also confirm that solving Model 2 yields good quality solutions much faster. Whereas the 118-bus case only shows an improvement of 0.1%, we reach up to 19% in others.

V. Conclusion

In this paper we outlined a method to decrease optimal dispatch costs by reconfiguring the network's topology, i.e. by transmission switching and bus splitting. This method and model, as opposed to prior research in this field, can account arbitrary substation configurations. We additionally outlined a technique to improve the solution process (due to the models computational complexity). Using modern software, we are able to provide optimal solutions for this highly complex problem. Our results show that significant improvements can be achieved in a competitive computational time frame. The improvements for the 118 bus case in Figure 4 are marginal, but Table I illustrates, that improvements of up to approx. 20% can be achieved for selected test cases. Considering N-1 security, modelling real-world substations or more elaborate computational methods will be directions of further work on the topic.

References

[1] E. B. Fisher, R. P. O'Neill, and M. C. Ferris, "Optimal transmission switching," *IEEE Transactions on Power Systems*, vol. 23, no. 3, pp. 1346–1355, 2008.

[2] H. Glavitsch, "Switching as means of control in the power system," *International Journal of Electrical Power & Energy Systems*, vol. 7, no. 2, pp. 92 – 100, 1985.

[3] G. Schnyder and H. Glavitsch, "Security enhancement using an optimal switching power flow," *IEEE Transactions on Power Systems*, vol. 5, no. 2, pp. 674–681, 1990.

[4] K. W. Hedman, R. P. O'Neill, E. B. Fisher, and S. S. Oren, "Optimal transmission switching with contingency analysis," *IEEE Transactions on Power Systems*, vol. 24, no. 3, pp. 1577–1586, 2009.

[5] C. Liu, J. Wang, and J. Ostrowski, "Static switching security in multi-period transmission switching," *IEEE Transactions on Power Systems*, vol. 27, no. 4, pp. 1850–1858, 2012.

[6] J. Villumsen and A. Philpott, "Investment in electricity networks with transmission switching," *European Journal of Operational Research*, vol. 222, no. 2, pp. 377 – 385, 2012.

[7] R. Saavedra, A. Street, and J. M. Arroyo, "Day-ahead contingency-constrained unit commitment with co-optimized post-contingency transmission switching," *IEEE Transactions on Power Systems*, pp. 1–1, 2020.

[8] K.Lehmann, A. Grastien, and P. V. Hentenryck, "The complexity of dc-switching problems," NICTA, Tech. Rep., 2014.

[9] S. Namchoat and N. Hoonchareon, "Optimal bus splitting for short-circuit current limitation in metropolitan area," in *2013 10th International Conference on Electrical Engineering/Electronics, Computer, Telecommunications and Information Technology*, 2013, pp. 1–5.

[10] Wei Shao and V. Vittal, "Corrective switching algorithm for relieving overloads and voltage violations," *IEEE Transactions on Power Systems*, vol. 20, no. 4, pp. 1877–1885, 2005.

[11] M. Heidarifar, M. Doostizadeh, and H. Ghasemi, "Optimal transmission reconfiguration through line switching and bus splitting," in *2014 IEEE PES General Meeting — Conference Exposition*, 2014, pp. 1–5.

[12] M. Heidarifar and H. Ghasemi, "A network topology optimization model based on substation and node-breaker modeling," *IEEE Transactions on Power Systems*, vol. 31, no. 1, pp. 247–255, 2016.

[13] *Schaltanlagen*. Berlin, Heidelberg: Springer Berlin Heidelberg, 2009, pp. 529–584.

[14] W. Shao and V. Vittal, "A new algorithm for relieving overloads and voltage violations by transmission line and bus-bar switching," in *IEEE PES Power Systems Conference and Exposition, 2004.*, 2004, pp. 322–327 vol.1.

[15] U. A. Bakshi and M. V. Bakshi, *Power System - II*, 1st ed. Technical Publications Pune, 2009.

[16] L. L. Grigsby, *Electrical Engineering Handbook*. CRC Press, 2000.