Online Demand Response for End-User Loads

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***Abstract*—Smart grids as digitalized electricity networks can provide many new capabilities such as managing an enormous number of distributed energy resources, supporting large quantities of renewable energy productions even in small-scales, as well as enabling demand side to participate more actively in demand response (DR) programs. In the depth of digital communication capabilities, alternative decision-making tools are needed for providing adequate solutions to satisfy the involved customers with the new reality: decisions have to be made fast (online) and with the scarce information about the future. However, the state-of-the-art on DR has been providing decision-making tools based on conventional optimization framework that are carried offline, while the real-time nature of most of DR programs requires online optimization approaches. In this regard, we present an online DR model for an end-user load that receives price information on real time and decides about the next action in a completely online fashion. Then, we present an algorithm based on the gradient descent method for solving the proposed DR model. The theoretical model and its applicability are presented and verified using numerical simulations. The results demonstrate the ability to reach considerable profits in a simple and easy-to-implement procedure with limited exogenous data and no information about future random prices.**

***Index Terms*—Demand Response, Online Convex Optimization, Uncertainty, Smart Grids.**

I. Introduction

The increase of demand and renewable generation, the development of new technologies and the modernization of societies is shifting conventional power systems towards a smart grids paradigm. This is especially more relevant at the distribution networks, close to the demand loads. Smart grid as a digitalized network, enables a two-way electricity and information flow for end-users and system operators [1], [2]. This opens a large set of possibilities to enable demand-side participation in power system through the mechanisms known as demand response (DR) [3]. However, DR problem in its most essential nature differs from traditional modeling structures of power systems operation, despite the extensive body of research proposing optimization models and methods that resemble the existing optimal operation of power systems. DR is essentially an online decision-making problem with limited data about the very random stochastic future. For instance, it is nearly impossible to model the vast number of underlying random processes (e.g., when an end-user decides to charge its cellphone) that are involved in the utility/cost of DR participants. Also, the decision-making process has to be done in an online fashion: from microseconds to a few minutes. Traditionally, offline optimization models for the decision-making under uncertainty requires uncertainty characterization about the future random events/parameters while at the same time, they demand significant computational time for providing a solution. In this paper, we propose an alternative online model for DR that requires minimal information about the future and it takes microseconds for providing a solution.

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A. Literature Review

Many researches have been done in last decade investigating the potential of DR for different applications in a smart grid [4]–[19]. In this regard, they have utilized different mathematical approaches to the problem modeling and solving methodologies. For instance, convex optimization is used in [4]–[6], also with some relaxation, the model proposed in [2] can be presented as a convex optimization. Dynamic programming is another common method applied in the literature related to DR [7], [8], considering each interval as a stage in dynamic programming. In [9] a forward dynamic programming algorithm is also utilized to assess the best evolution from a state at a given time to another. DR may comprise some game actions among the related end-users or between the power system operator and users. In this order, game theory approaches have been used to address this problem as studied in [10] and [11]. DR models also deal with the uncertainty of future, and unknown parameters such as electricity price and demand amount leading to the introduction of stochastic methods in the literature of DR [12]–[14]. These papers capture the uncertainties through the generation of many possible scenarios and running the model considering an expectation of the objective function.

All the above-mentioned approaches for optimal DR and related energy management studies are performed in an *offline fashion* most of which presuming perfect predictions for the renewables, demands, and market, which in practice is hard to accomplish considering the intermittency and unpredictability of renewable productions, the temporal uncertainty in controllable loads as well as the real-time pricing randomness [15], [16]. On the other hand, uncertainty consideration dealt with stochastic models which mostly employ different scenarios for outcomes of uncertain parameters, due to high numbers of scenarios can be computationally expensive and time-consuming. Even in robust optimization models which are also used in DR related studies [17], [19], there is a requirement for prior knowledge about the uncertainties for building a confidence interval or a box-like uncertainty set. To consider all possible scenarios within the set, these models may provide results that are contemplated too conservative [20].

It should be noted that although these approaches consider the uncertainties, they still need certain forecasting and usually do not adapt to real-time changes in the environment, making them impotent when it comes to online decision making [21], [22]. Indeed, some DR programs belong to the *online decision making* categorization. In the online decision making, there is a sequence of optimizations that take place separately for each step or time. The online optimization differs from the other offline methods, as in the online optimization decision maker decides before the realization of the unknown parameter, then, when the information is revealed, decision maker understands the loss/profit for that period and updates his next decision accordingly [23].

B. Model Framework

In this study, we present a simple model of an online demand response. We consider a DR program similar to one presented in the [17]. We assume that a customer with the capability to control its electric consumption with ramp limits receives price signals at the beginning of each time slot and decides its energy consumption. In the proposed model not only data is revealed in an online fashion but also decision making is considered to match this feature of non-anticipativeness where decision is taken prior to disclosure of data and without any previous knowledge or prediction. Also, the decision-making is done only for that period and not for the others.

C. Paper Contributions and Organization

The main contribution of this work is the proposal of an online DR model for end-user loads. Contrary to the (deterministic or stochastic) offline framework where decisions are optimized for the entire time horizon of study, our framework relies on sequential decisions making with no distributional assumptions about the uncertainty and no assumption about the uncertain sequence of cost functions. We show that the online DR belongs to the class of online convex optimization (OCO). Then, we present an algorithm based on gradient descent methods to solve the online DR. Performance is compared against a robust offline model [17]. In summary, we show that the proposed model and algorithm is simple and natural for handling arbitrary sequence of uncertainties in DR decision-making.

The rest of the article is organized as follows. Section II describes the mathematical model. The numerical studies are presented in section III and section IV covers the conclusion of the paper.

II. Mathematical Model

A. Online Convex Optimization Preliminaries

In an online optimization, at each stage of a repeated process, t, an online agent is assumed to make a choice, x_t, among the set of possible actions, $\mathcal{K}$. At the beginning of each stage, the result of the chosen action is unknown for the agent. After committing to a specified choice the agent would incur a loss, $f_t(x_t)$, consequently receiving a feedback from his action which is used to decide the next choice of the upcoming stage, x_{t+1}.

If the loss function, $f_t(x_t)$ is convex in x_t, the online optimization will be in the category of the OCO[1]. The optimizing agent is seeking to minimize his regret which is the gap between cumulative loss based on his decisions and best decisions in hindsight, x^*, in the total optimization time, T. The regret is described in (1).

$$R_T = \sum_{t=1}^{T} f_t(x_t) - \sum_{t=1}^{T} f_t(x^*) \tag{1}$$

It is clear that regret at time T, R_T, is greater than zero. Also, it is completely dependent on the algorithm/policy chosen at each stage t: how x_{t+1} is updated. Ideally, if T is big enough, the average regret should tend to zero for a good algorithm.

One of the most popular algorithms in the literature of the OCO is *online gradient descent* (OGD). The reference [24] has shown that this algorithm achieves regret of $\mathcal{O}(\sqrt{T})$ when the set that x_t belongs to is also convex, and the loss function is Lipschitz continuous[2] within the set domain.

OGD method is introduced in the Algorithm 1. In this algorithm, at each stage, the decision is updated according to the gradient of the loss function, $\nabla f_t(x_t)$. If the resulted decision, y_{t+1}, is not within the bounds of the convex decision set, $\mathcal{K}$, it will be projected back to the set. Notice that the η parameter (step size) can be changed in each time, but here, we have considered it fixed for the sake of simplicity.

Algorithm 1: Online Gradient Descent

```
Initialize x_1 ∈ K, η ∈ R^+
for t = 1 to T do
    Compute losses f_t(x_t) for the selected x_t
    Compute: y_{t+1} = x_t − η∇f_t(x_t)
    Project and update: x_{t+1} = ∏_K(y_{t+1})
end
```

B. Offline Demand Response Model

For DR program we consider a case similar to the one presented in [17]. In this case, consumer has the ability to change its consumption in the upcoming hours limited by some ramp constraints. Customer has a utility function related to the amount of energy that is consumed but customer needs to buy this energy with a price that is communicated at each

[1] A function $f(x)$ is convex if it satisfies $f(y) \geq f(x) + \nabla f(x)(y-x)$, $\forall x, y \in \mathbb{R}^n$.

[2] A function $f(x)$ is Lipschitz continuous by parameter D if $\|f(x) - f(y)\| \leq D\|x - y\|,\ x, y \in \mathbb{R}^n$.

period t (hourly in our case) by the grid. If we assume perfect prediction of electricity prices for the whole day, the offline model for the optimal DR is described by

$$H(\boldsymbol{\lambda}, \mathbf{u}) = \min_{\Xi} \sum_{t=1}^{T} (\lambda_t e_t - u_t e_t) \tag{2a}$$

s.t.:

$$\sum_{t=1}^{T} e_t \geq E_T \tag{2b}$$

$$e_t = \frac{d_{t+1} + d_t}{2}, \qquad \forall t = 1, \ldots, T-1 \tag{2c}$$

$$d_{t+1} - d_t \leq r^{up}, \qquad \forall t = 1, \ldots, T-1 \tag{2d}$$

$$d_t - d_{t+1} \leq r^{dn}, \qquad \forall t = 1, \ldots, T-1 \tag{2e}$$

$$d_t^{min} \leq d_t \leq d_t^{max}, \qquad \forall t = 1, \ldots, T \tag{2f}$$

where decision variables of this problem are in the set $\Xi = \{\mathbf{e} = [e_t,\ t = 1, \ldots, T], \mathbf{d} = [d_t,\ t = 1, \ldots, T]\}$.

The above formulation explores optimal values for the electricity consumption in each period according to the enforced ramp limits, while at the same time fulfilling the minimum amount of energy consumption for the total period of T. Here function $H(\boldsymbol{\lambda}, \mathbf{u})$ is expressing the total cost of the customer dependent on the electricity price and the utility at each hour.

The objective of the problem (2) consists of minimizing the difference between the cost of buying energy and utility acquired by consuming energy in each period. Here, λ_t and u_t are the price of electricity and utility of customer at time t respectively. Constraint (2b) is representing the minimum amount of consumption at time T. In (2c) the relationship between the energy at time t and demand is shown. Energy consumed at time t is considered to be the average of two demand amount at the beginning and at the end of a time period with one hour length. Equations (2d) and (2e) are the ramp up and ramp down limits. Similar to a power generation unit, variations in demand can be restricted. These two constraints ensure that, in each time period demand does not change more than a predefined amount. Note that these values are set by customer and can be changed during an optimization period. Finally, (2f) represents the bounds of a set that demand amount can be selected from.

It is worth to remark that the model described above assumes that $\boldsymbol{\lambda}$ and $\mathbf{u}$ are known a priori. Thus, the problem (2) is classified as a deterministic linear programming (LP) problem. It is possible to create an offline stochastic version of this model where it is assumed, for example, that $\boldsymbol{\lambda}$ is uncertain. This uncertain parameter can be represented by a set of scenarios [18] or an uncertainty set [17] known a priory. That is, we have complete or partial knowledge of the stochastic process representing $\boldsymbol{\lambda}$. Multi-stage stochastic or robust optimization frameworks could be used to derive the optimal decisions for T periods in an offline fashion. On the other hand, in the online optimization framework, the parameters are uncertain and we have information about the observed performance of certain decisions x_t at the end of each interval t. Therefore, there is no assumption on the statistical properties of the uncertain parameter neither in the sequence.

C. Online Demand Response Framework

If we take a look at the optimization introduced in the previous section we can see that it is far from reality. Because there are many imposed assumptions on the uncertain parameters that are weakly supported. For instance, the exact amount of cost and utility which will be revealed only in the future, at the end of each interval. Indeed, the structure of the introduced DR program and many others are very unique because of the variability of parameters in the optimization models. One may easily see that data will reveal itself at each time and so does the loss function of the customer. In this regard, we would like to address the DR problem (2) as a fully online optimization without using any historical information or adding additional complexity.

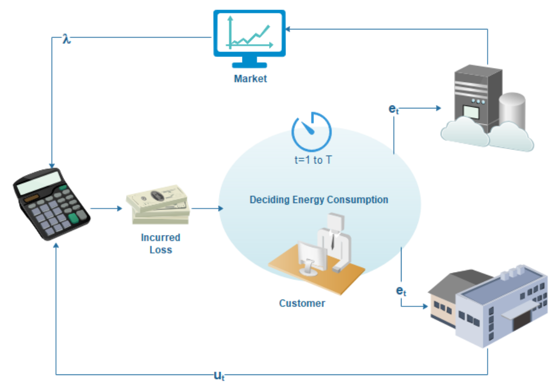

Figure 1. The online decision making process.

Thus, in the online DR, the customer does not need to rely on the previous information or any stochastic or robust model that adds complexity, data related and time-consuming operations. The customer makes a decision about the amount of energy consumption for the next time and when the price will be communicated and the utility is determined, the incurred loss can be calculated. Then, according to the algorithm that we present in this section, customer updates the next action meaning the amount of energy that will be consumed in the upcoming period. This is done for an entire period which in this case will be a whole day. For example, if data is received hourly, the sequence of the optimization is done in the time starting with $t = 1$ and ending with $T = 24$. It is clear that the optimization can be done with more steps if the data is revealed in faster versions, for instance, every 5 seconds resulting in $T = 17280$. Observe that the online model use information of the current period t only.

The simplified process of the decision making is demonstrated in Fig. 1 for better clearance of the subject. As depicted in this figure, customer decide its consumption for the next hour and this value is considered the committed power. Then, price and utility regarding this power are determined and communicated to the customer. In this regard, loss function is revealed (in this period) that can be used in further decision making of customer.

D. Solution Methodology

In this section, we present an online algorithm to solve the aforementioned online DR problem. By inspecting the loss function that occurs to the customer at each step, and the constraints of the optimization, it is clear when considering u_t as a linear function or a fixed amount as done in [17], the loss function, f_t, and related constraints are linear meaning that they are convex and the concept of OCO is applicable here.

However, the DR model is different as it is constrained by different conditions related to the amount of energy and demand at each hour. We separate these constraints and divide them into two different groups: first, the constraints that should not be violated at time t, and second, the ones that should not be violated only at the end of time T. It is easy to see that ramp constraints belong to the first group and minimum energy consumption, for example, during a day belongs to the latter.

We first consider the second group of the constraints and present the proper methodology and algorithm accordingly, then we also exhibit how to take into account the ramp constraints.

In the DR model there is constraint (2b) stating that consumption of energy during the day should not be less than a predefined amount. This condition only holds at the last period and is not a hard limit at each time, thus, we propose to go from a constrained optimization to an unconstrained one using the penalty term in the objective function which in here is replacing the original loss function $f_t(x_t)$ by the modified one $\hat{f}_t(x_t)$. It is generally defined by

$$\hat{f}_t(x_t) = f_t(x_t) + \gamma g_i(x_t), \quad i = 1, \cdots, N \tag{3}$$

where $\hat{f}_t(x_t)$ is the new loss function that next decision is updated according to its gradient, γ is a fixed multiplier related to the penalization, and $g_i(x)$ is the i^{th} constraint of the basic optimization, (2b) in this case.

However, the OGD algorithm, as it is shown in [25], can be enhanced for finding efficient solution by adding a new regularization term to the loss function (3). Subsequently, we utilize the new loss function presented in (4), [26].

$$\mathcal{L}_t(x_t, \gamma) = f_t(x_t) + \gamma g(x_t) - \frac{\delta}{2}\gamma^2 \tag{4}$$

Indeed $\mathcal{L}_t(x, \gamma)$ is a new extended loss function that we use when updating the next-step decisions. The regularization term $\frac{\delta}{2}\gamma^2$ is added to prevent that γ becomes very big at step, t. Using OGD-based algorithm with introduced function, as demonstrated in [26], can achieve sub-linear regrets.

In our optimization model we still need to include ramp constraints. Despite the previous case, ramp constraints impose hard limits on the decision making process. Therefore, they should not be violated at any step. In this regard, we propose that instead of projecting back to the initial set of feasible solutions, $\mathcal{K}$, which defines the basic decision set, considering a smaller set at each step inside the previous one named by $\mathcal{B}_t$. At each step after making the final decision according to the gradient of $\mathcal{L}_t(x_t, \gamma)$, regarding ramps hard limits, the result would be projected back to the feasible time-dependent set of $\mathcal{B}_t \subseteq \mathcal{K}$. In this way, the constraints never get violated for any time step in the whole optimization. We conclude the above explanation in ALGORITHM 2.

Algorithm 2: OGD-based Demand Response

Initialize $e_1 \in \mathcal{K}, \gamma_1 \in [0, +\infty), \eta \in \mathbb{R}^+, \mu \in \mathbb{R}^+$
for $t = 1$ to T **do**
  Observe λ_t
  Compute $f_t(e_t) = (\lambda_t - u_t)e_t$
  Compute $y_{t+1} = e_t - \eta \nabla_e \mathcal{L}_t(e_t, \gamma_t)$
  Compute $z_{t+1} = \gamma_t - \mu \nabla_\gamma \mathcal{L}_t(e_t, \gamma_t)$
  Project and update: $e_{t+1} = \prod_{\mathcal{B}_{t+1}}(y_{t+1})$
  Project and update: $\gamma_{t+1} = \prod_{[0,+\infty)}(z_{t+1})$
end

In the DR program, the customer decides the energy of the next hour, therefore $x_t = e_t$. After deciding on the energy of the next hour, the price of energy and the utility of customer are known. Assuming a fixed utility at each hour, the loss function can be calculated for every hour as, $(\lambda_t - u_t)e_t$. Thus, the extended loss function is described as follows:

$$\mathcal{L}_t(e_t, \gamma) = (\lambda_t - u_t)e_t + \gamma_t\left(\frac{E_T}{T} - e_t\right) - \frac{\delta}{2}\gamma_t^2 \tag{5}$$

Then, in updating part of the algorithm, first energy at t is calculated according to gradient of $\mathcal{L}_t(e_t, \gamma_t)$ then projected back to $\mathcal{B}_{t+1}$ defined by ramp limits on the demand at time t. Finally, γ_t is also updated providing the long-term constraint conditions at T. It should be noted that γ_t is selected from the set $[0, +\infty)$.

III. Numerical Study

In this section, we analyze the performance of our model and the proposed algorithm. The results are also compared with a base case and algorithm, which combines rolling window and robust optimization, presented in detail in [17]. Data of the customer is depicted in Table I, the prices for the upcoming day in Table II and the initial demand of 1.5 MW.

A. Utility Analysis with Real Data

The DR model of [17] uses the real-time price data at each hour and runs a robust optimization for the upcoming time-periods. Considering the uncertainty of price, in expectation, higher values for the budget of uncertainty Γ, result in higher income or utility for the customer. Nevertheless, in reality, the optimum value is unknown and determining it beforehand is not possible. This value typifies the decision of the customer on the level of robustness that he/she desires.

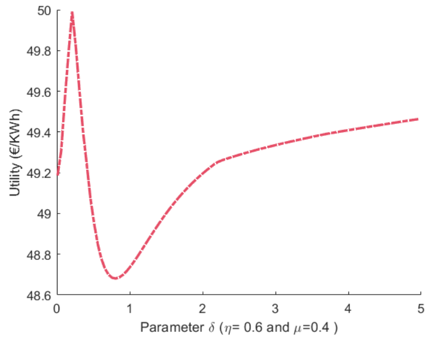

Figure 4. The utility of customer with variation of δ.

TABLE I
CUSTOMER DATA

Information	Quantity
Maximum demand at each hour	3 MW
Minimum demand at each hour	0 MW
Minimum daily consumption	15 MWh
Ramping up limit	1 MW/h
Ramping down limit	1 MW/h
Customer utility	41.5 €/MWh

TABLE II
ENERGY PRICES €/MWH

Time	Price	Time	Price
1	44.80	13	45.61
2	41.03	14	45.42
3	36.10	15	39.28
4	33.00	16	41.16
5	33.00	17	42.01
6	36.46	18	43.00
7	43.01	19	41.16
8	47.05	20	41.63
9	46.06	21	42.00
10	45.51	22	41.16
11	46.06	23	41.87
12	44.50	24	36.81

In the proposed OGD-based algorithm there are also step sizes needed to be defined. These steps can be modified at each time but, to simplify, we have considered fixed and non-updatable step sizes. The result of optimizations regarding the input data is presented in Table III. As can be seen, the utility that customer acquires at the end of the day depends on Γ in robust optimization and step sizes in the online algorithms. Here, we have fixed $\mu = 0.2$ and $\delta = 0.05$.

The most important thing about the acquired results is that despite using no additional data and predictions and only running the optimization for one time at each step, still the online algorithm is performing satisfactory and the acquired utility in many cases is comparable to the robust real-time rolling window optimization. Besides, in the online algorithm the proper step size can be chosen more easily in comparison with the budget of uncertainty where its optimal result happens in only one value, $\Gamma = 45$, unknown to the customer prior to the last time of the optimization.

B. Impact of Step Size Parameters

In this part, we investigate the impact of step sizes on the evolution of OCO to reach its optimum value for the utility of the customer. In this order, first, we fix the other two parameters and then run the optimization for different values of the remaining parameter.

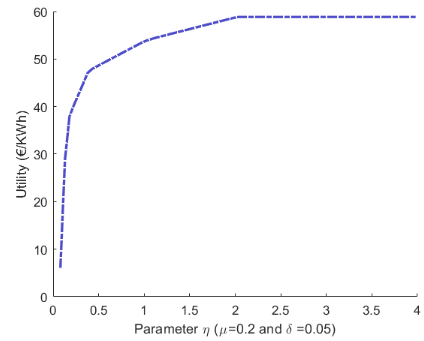

Figure 2. The utility of customer with variation of η.

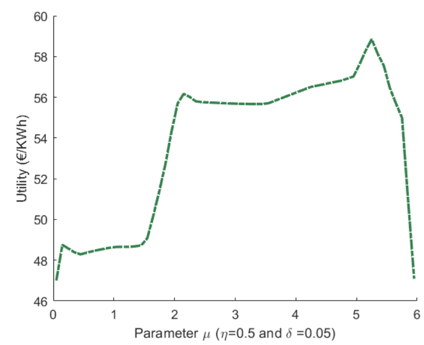

Figure 3. The utility of customer with variation of μ.

In Fig. 2, 3 and 4, the utility of customer at the end of the day is demonstrated regarding variation in parameter η, μ, and δ respectively. η is the main step which is directly related to the updated value of energy for the next time-period. This is also understandable from Fig. 2 as the utility value varies noticeably with changes in this parameter. In these figures we intentionally have chosen low values for the fixed parameters to see the variation of utility related to the remaining parameter. The other parameter is μ which is related to updating the next γ. As it can be seen, even if η is not chosen properly, to some extent, it can be compensated through proper choice of μ. These figures also suggests that selecting big values for these parameters does not necessary leads to a better result.

C. Computational Complexity

We would like to report a simple observation of the time consumption for two optimization models: the proposed OCO and the real-time robust rolling window. We executed both models for 100 days on a server computer with CPU 2.4 GHz and 256 GB RAM. We took 6 values of Γ only for the robust

TABLE III
CUSTOMER UTILITY (€) FOR ROBUST AND ONLINE OPTIMIZATION CONSIDERING DIFFERENT UNCERTAINTY BUDGET AND STEP SIZES

Γ	Utility (RO)	Utility (OCO)	η
0	-2.49	15.42	0.1
5	-2.13	41.28	0.25
10	-1.87	47.62	0.4
15	-9.7	49.17	0.55
20	0.22	50.7	0.7
25	1.24	52.21	0.85
30	2.06	53.17	1
35	6.95	54.6	1.15
40	72.24	55.34	1.3
45	77.07	56.07	1.45
50	66.81	56.8	1.6
55	51.03	57.5	1.75
60	56.68	58.27	1.9
65	64.88	58.86	2.05
70	69.73	58.86	2.2
75	69.73	58.86	2.35
80	69.73	58.86	2.5
85	69.73	58.86	2.65
90	69.73	58.86	2.8
95	69.73	58.86	2.95
100	69.73	58.86	3.1

model and we compared it with running the OCO for 710 different possible step sizes.

The OCO is surprisingly fast and manage to simulate 71000 iterations (hourly energy output updates) only in 0.29 second while the robust model does 600 repetitions of optimization in 5985 seconds. It is clear, if development of the infrastructures and technologies move smart grids toward faster communication of data, the classical offline optimization approaches would not suffice regarding the time needed to finish the required optimization.

IV. CONCLUSION

Demand response programs have been in the center of attention since the introduction of new communication technologies in power grids allowing active participation of end-users. But availability of data does not always take place prior to running decision-making optimization. For instance, in the case of the real-time price, it will be revealed once for only one interval. In this regard, we were intrigued to rethink demand response programs in an online fashion, we proposed an alternative online algorithm to address the DR problem. The result showed that our proposed approach can achieve considerable results in a very fast time, despite not seeking any exogenous data or utilizing prediction methods.

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