Electricity and Gas Network Expansion Planning: an ADMM-based Decomposition Approach

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***Abstract*—This paper presents a model for the electricity and gas expansion planning (E&GEP) problem. The gas network is modeled by non-linear equations based on the steady-state physical description of the gas system. The electricity network is modeled by the linearized power flow equations. Investment in new transmission lines, pipelines, compressors, and possible gas source connections are considered in our model. A decomposition method based on the alternating direction method of multipliers (ADMM) is proposed to solve this problem. We introduce a linear reformulation for the quadratic coupling constraint between electricity and gas networks. The ADMM algorithm can guarantee the gas and electricity system's data privacy while achieving the benefits of the co-planning work. The proposed method is applied in the co-planning of the IEEE 14-bus and the Belgian gas networks. An extensive number of numerical studies were performed for validating the proposed approach.**

***Index Terms*—ADMM, Planning, Decomposition, Electricity network, Natural gas.**

I. Introduction

Nowadays, natural gas is one of the main sources of electricity generation in several countries and its demand has grown very fast in recent years. This growth is accompanied by the increasing electricity demand that pushed by the electrification of several sectors, such as cleaner transportation for a more sustainable energy supply. Gas networks are responsible for supplying residential and industrial gas consumers. Therefore, both the electricity and natural gas networks are of particular importance in energy services. Meanwhile, an increasing number of equipment, especially gas-fired generators, are invested and put into use which strengthen coupling between these two systems. Hence, the electricity and gas expansion co-planning problem becomes an object of study to understand potential challenges and opportunities of joint planning and operation of these two sectors that traditionally have remained decoupled [1], [2]. During the recent years, new research initiatives on the combined planning of power and gas networks has intensified. For instance, the authors in [3] proposed an optimization-based model to minimize investment costs including equipment placement and installation scheduling for power and gas networks. Similarly, [4] presented an optimal network planning for both systems. The joint gas and electricity network expansion planning has been also studied in real systems such as western Denmark [5], Queensland state in Australia [6], and Great Britain [7].

It should be noted that the most important issue on the planning and operating of the above-mentioned networks is the complexity in modeling the nonlinear gas flow equations [8]. Thus, the combined planning problem usually results as mixed integer non-linear programming (MINLP). To overcome non-linearities issues, various methods have been applied such as mathematical approximation [9], linearization [10] and decomposition [11]. For example, the authors in [10] approximated the non-linear model by a linear programming one.

The E&GEP problem formulation is introduced firstly in this paper. Then a decomposition method based on the ADMM algorithm is proposed to solve the problem, which offers a unique advantage to get rid of the difficulties brought by information barriers between electricity and gas system. That is to say, the operators of electricity and gas network do not need to share their system information including energy demand or reserve condition. In this case, they only share information related with the coupling equipment between both systems (gas-fired units). Using the ADMM algorithm, the optimization problem is broken into smaller fragments. Each part is easier to solve [12] and parallelizable. This algorithm has no convergence guarantee in the cases of problems with integer variables. However, in our extensive numerical tests it has been shown that empirically, the ADMM has good performance and convergence properties.

The rest of this paper is organized as follows. The mathematical models of the combined power and gas planning are introduced in Section II. After this section, the decomposition methodology comes in Section III. Sections IV and V are devoted to present numerical simulations and conclusions, respectively.

II. Mathematical Models

The E&GEP problem consists of finding the new assets updates from gas and electricity networks aiming to minimize

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the overall investment and operational costs of both networks while meeting electricity and gas systems constraints. Thus, the E&GEP model can be described as follows

$$\begin{aligned} \min \quad & \beta\left(C^{GEP}+C^{EEP}\right)+\left(C^{GO}+C^{EO}\right) \qquad (1)\\ \text{s.t.:} \quad & \text{Gas Network (2)}\\ & \text{Electricity Network (3)}\\ & \text{Electricity–Gas Coupling (4)} \end{aligned}$$

where the objective function is composed by C^{GEP} representing the total gas system expansion planning cost, including installation of new gas sources, pipelines and compressors; C^{EEP} is the total electric network expansion planning cost, including investment on new generation units and electric lines; C^{GO} is the gas system operation cost; C^{EO} is the electric network operation cost; and the symbol β is the weighting factor between system investment cost and operation cost, which can be set differently depending on the lifetime of the invested cost and the period of the involved operating cost. The set of electricity and gas network constraints are defined in the following sections.

A. Gas System Expansion Planning Constraints

Gas network mainly comprises gas wells, pipelines, compressors, valves and gas consumers. Based on the gas expansion planning model proposed in [13], constraints in the gas system expansion planning model can be expressed as in (2).

V_g represents all vertices (nodes) set in the gas system. E_g represents all edges set in the gas model, including existing pipelines set E_{gep}, existing pipelines with compressors set E_{gec}, new pipelines candidates set E_{gnp}, new compressors candidates on existing pipelines set E_{gncep} and new compressors on new pipelines set, E_{gncnp}. E_{gncep} group will be divided into two groups, E_{gncep1} representing the candidate compressors that can be installed on the pipeline where there is no parallel pipes and E_{gncep2} for the compressors set which will be installed along with the existing parallel pipelines. Meanwhile, E_{gncnp} will be classified into three groups, E_{gncnp1} stands for the new compressors on new pipelines set where there exists a parallel element in E_{gec} set; E_{gncnp2} contains the new compressors on new pipelines set in parallel to an element in E_{gncep}; E_{gncnp3} relates with the new compressors on new pipelines set without parallel pipelines.

Equation (2a) is formed based on network flow continuity in the gas system, where $\phi_{a,ij}$ represents the gas flow in pipeline a connection node i to node j. $\phi_{a,ij}$ is positive if the gas flow is flowing from node i to node j, and negative otherwise. GW_i represents gas sources set on node i. s_k represents the quantity of gas supply from gas source k. b_k^s represents binary variables for gas sources k; For existing gas source, b_k^s will be always equal to 1. $d_{e,i}$ represents gas demand on node i for gas-fired generator units; $D_{o,i}$ represents gas demand on node i for other uses.

In (2b) for existing pipelines and (2c) for pipelines candidates, steady-state gas flux are depicted as in [13]. b_a^p represents binary variables for new pipelines candidates in the gas system; $y_{a,ij}^+$ and $y_{a,ij}^-$ are defined as binary direction variables for all pipes, when the gas flux in pipe a flows

CONSTRAINTS SET 1 Gas System Model

$$\sum_{j:(j,i)\in E_g}\phi_{a,ji}+\sum_{k\in GW_i}s_k b_k^s=\sum_{j:(i,j)\in E_g}\phi_{a,ij}+d_{e,i}+D_{o,i},\quad \forall i\in V_g \qquad (2a)$$

$$(y_{a,ij}^+-y_{a,ij}^-)(\pi_i-\pi_j)=w_a\phi_{a,ij}^2,\quad \forall a_{ij}\in E_{gep} \qquad (2b)$$

$$b_a^p(y_{a,ij}^+-y_{a,ij}^-)(\pi_i-\pi_j)=w_a\phi_{a,ij}^2,\quad \forall a_{ij}\in E_{gnp} \qquad (2c)$$

$$(1-y_{a,ij}^+)(\underline{\Pi}_i-\overline{\Pi}_j)\le\pi_i-\pi_j\le(1-y_{a,ij}^-)(\overline{\Pi}_i-\underline{\Pi}_j),\quad \forall a_{ij}\in E_{gep} \qquad (2d)$$

$$b_a^p(1-y_{a,ij}^+)(\underline{\Pi}_i-\overline{\Pi}_j)\le b_a^p(\pi_i-\pi_j)\le b_a^p(1-y_{a,ij}^-)(\overline{\Pi}_i-\underline{\Pi}_j),\quad \forall a_{ij}\in E_{gnp} \qquad (2e)$$

$$\pi_i\alpha_c^l-(1-y_{a,ij}^+)(\overline{\Pi}_i\alpha_c^l-\underline{\Pi}_j)\le\pi_j\le\pi_i\alpha_c^u+(1-y_{a,ij}^+)(\overline{\Pi}_j-\underline{\Pi}_i\alpha_c^u),\quad \forall a_{ij}\in E_{gec} \qquad (2f)$$

$$\pi_j\alpha_c^l-(1-y_{a,ij}^-)(\overline{\Pi}_j\alpha_c^l-\underline{\Pi}_i)\le\pi_i\le\pi_j\alpha_c^u+(1-y_{a,ij}^-)(\overline{\Pi}_i-\underline{\Pi}_j\alpha_c^u),\quad \forall a_{ij}\in E_{gec} \qquad (2g)$$

$$-b_a^cM\le(y_{a,ij}^+-y_{a,ij}^-)(\pi_i-\pi_j)-w_a\phi_{a,ij}^2\le b_a^cM,\quad \forall a_{ij}\in E_{gncep1}\cup E_{gncep2}\cup E_{gncnp1} \qquad (2h)$$

$$-M(b_a^c-b_a^p+1)\le(y_{a,ij}^+-y_{a,ij}^-)(\pi_i-\pi_j)-w_a\phi_{a,ij}^2\le M(b_a^c-b_a^p+1),\quad \forall a_{ij}\in E_{gncnp2}\cup E_{gncnp3} \qquad (2i)$$

$$\pi_i\alpha_c^l-(2-y_{a,ij}^+-b_a^c)(\overline{\Pi}_i\alpha_c^l-\underline{\Pi}_j)\le\pi_j\le\pi_i\alpha_c^u+(2-y_{a,ij}^+-b_a^c)(\overline{\Pi}_j-\underline{\Pi}_i\alpha_c^u),\forall a_{ij}\in E_{gncep}\cup E_{gncnp} \qquad (2j)$$

$$\pi_j\alpha_c^l-(2-y_{a,ij}^--b_a^c)(\overline{\Pi}_j\alpha_c^l-\underline{\Pi}_i)\le\pi_i\le\pi_j\alpha_c^u+(2-y_{a,ij}^--b_a^c)(\overline{\Pi}_i-\underline{\Pi}_j\alpha_c^u),\forall a_{ij}\in E_{gncep}\cup E_{gncnp} \qquad (2k)$$

$$(1-b_a^c)(1-y_{a,ij}^+)(\underline{\Pi}_i-\overline{\Pi}_j)\le(1-b_a^c)(\pi_i-\pi_j)\le(1-b_a^c)(1-y_{a,ij}^-)(\overline{\Pi}_i-\underline{\Pi}_j),\forall a_{ij}\in E_{gncep}\cup E_{gncnp} \qquad (2l)$$

$$b_a^c=b_{\tilde{a}}^c,\quad \forall a_{ij},\tilde{a}_{ij}\in E_{gncep2} \qquad (2m)$$

$$b_a^c=b_a^p,\quad \forall a_{ij}\in E_{gncnp1} \qquad (2n)$$

$$b_a^c=b_a^pb_{\tilde{a}}^c,\quad \forall a_{ij},\tilde{a}_{ij}\in E_{gncnp2} \qquad (2o)$$

$$b_a^c\le b_a^p,\quad \forall a_{ij}\in E_{gncnp3} \qquad (2p)$$

$$\phi_{a,ij}\le(1-y_{a,ij}^-)\sum_{i\in V_g}\sum_{s_k\in GW_i}s_kb_k^s,\quad \forall a_{ij}\in E_g \qquad (2q)$$

$$\phi_{a,ij}\ge(y_{a,ij}^+-1)\sum_{i\in V_g}\sum_{s_k\in GW_i}s_kb_k^s,\quad \forall a_{ij}\in E_g \qquad (2r)$$

$$y_{a,ij}^++y_{a,ij}^-=1,\quad \forall a_{ij}\in E_g \qquad (2s)$$

$$\pi_i\le c_i\overline{\Pi}_i,\quad \forall i\in V_g \qquad (2t)$$

$$\underline{\Pi}_i\le\pi_i\le\overline{\Pi}_i,\quad \forall i\in V_g \qquad (2u)$$

$$\underline{S}_i\le s_i\le\overline{S}_i,\quad \forall i\in V_g \qquad (2v)$$

$$\underline{D}_{e,i}\le d_{e,i}\le\overline{D}_{e,i},\quad \forall i\in V_g \qquad (2w)$$

from node i to node j, then $y^+_{a,ij} = 1$, $y^-_{a,ij} = 0$; otherwise, $y^+_{a,ij} = 0$, $y^-_{a,ij} = 1$. π_i and π_j represent the square number of pressure on node i and node j, respectively, being connected by pipeline a; w_a represents pipeline coefficient related to the property of each pipe, which can be calculated as in [14].

In (2d) and (2g), $\overline{\Pi}$ and $\underline{\Pi}$ are the acceptable maximum and minimum values of the squared pressure on nodes in the system; α^l_c and α^u_c represent the lower and upper compression coefficient bound of the compressor, respectively. These four constraints describe the pressure difference condition between the two nodes on a pipe or pipe with compressor.

In (2h) and (2i), b^c_a represents binary variables for candidate compressors on pipeline a; M is a large enough constant. These two constraints ensure that the relationships between the gas flow and pressure will be satisfied only when the pipeline exist and the compressor not installed. Equations (2j)–(2l) has the same function as (2e)–(2g), showing how the pressure difference is related to the gas flow direction and the installed facilities on the pipe. The relationship between the installation decisions on parallel pipelines is described in (2m) and (2o). Equations (2n)–(2p) show that the compressor installation decision can only be made after the pipeline exists.

Equations (2q)–(2s) are obtained by the definition of $y^-_{a,ij}$ and $y^+_{a,ij}$. By (2t), the node pressure is enforced to be 0 for each isolated gas node in the system. c_i represents whether node i is being connected with gas network. When the node is connected, $c_i = 1$; otherwise, $c_i = 0$. The lower and upper bound of squared pressure, gas supply and gas demand for gas-fired units are shown in (2u)–(2w).

B. Electricity Network Expansion Planning Constraints

The linearized DC power flow equations are used for representing the electrical transmission network. The constraints in the electricity network expansion planning problem are summarized in (3).

Equation (3a) is the electricity network flow continuity equations, where V_e represents all the nodes set in the electricity network. E_e represents all the edges set in the electricity network, including existing electric lines set, E_{eel}, and new candidate lines set, E_{enl}. $f_{h,mn}$ represents the power flow of line h going from node m to node n. GP_n and OP_n are the sets of gas-fired power units and other power generators, respectively, installed on node n. $p_{o,z}$ and $b^{P_o}_z$ are the power output of generation units z using other sources and its related binary variables installed on node n. $p_{g,z}$ and $b^{P_g}_z$ are the power output of gas-fired units q and its related binary variables installed on node n. L_n is the electricity load on node n.

S_B represents the base value of power in the system. θ_m represents the voltage phase angle of node m. $x_{h,mn}$ represents the reactance of line h. b^l_h represents the binary variables for new candidate line h. $\overline{F}_{h,mn}$ represent the electricity line capacity. $\underline{P}_{g,z}$, $\overline{P}_{g,z}$, $\underline{P}_{o,z}$ and $\overline{P}_{o,z}$ are the lower and upper output limits for gas-fired and other source-supplied generators. M is a large enough constant. $\underline{\theta}_n$ and $\overline{\theta}_n$ are the lower and upper acceptable range of voltage angle on node n; $\theta_{sb} = 0$ sets the voltage phase angle of the slack bus to 0.

CONSTRAINTS SET 2 Electricity System Model

$$\sum_{m:(m,n)\in E_e} f_{h,mn} + \sum_{z\in OP_n} p_{o,z} b^{P_o}_z + \sum_{z\in GP_n} p_{g,z} b^{P_g}_z = \sum_{m:(n,m)\in E_e} f_{h,nm} + L_n, \quad \forall n \in V_e \tag{3a}$$

$$f_{h,mn} = \frac{S_B}{x_{h,mn}}(\theta_m - \theta_n), \quad \forall h_{mn} \in E_{eel} \tag{3b}$$

$$-\overline{F}_{h,mn} \le f_{h,mn} \le \overline{F}_{h,mn}, \quad \forall h_{mn} \in E_{eel} \tag{3c}$$

$$-M(1-b^l_h) \le f_{h,mn} - \frac{S_B}{x_{h,mn}}(\theta_m - \theta_n) \le M(1-b^l_h), \quad \forall h_{mn} \in E_{enl} \tag{3d}$$

$$-b^l_h \overline{F}_{h,mn} \le f_{h,mn} \le b^l_h \overline{F}_{h,mn}, \quad \forall h_{mn} \in E_{enl} \tag{3e}$$

$$b^{P_o}_z \underline{P}_{o,z} \le p_{o,z} \le b^{P_o}_z \overline{P}_{o,z}, \quad \forall z \in OP \tag{3f}$$

$$b^{P_g}_z \underline{P}_{g,z} \le p_{g,z} \le b^{P_g}_z \overline{P}_{g,z}, \quad \forall z \in GP \tag{3g}$$

$$\underline{\theta}_n \le \theta_n \le \overline{\theta}_n, \quad \forall n \in V_e \tag{3h}$$

$$\theta_{sb} = 0 \tag{3i}$$

C. Electricity–Gas Coupling Constraints

In our paper, gas-fired units are considered as the coupling components between electricity network and gas system [15]. The gas demand $d_{e,q}$ needed for gas-fired unit is coupled with its electric power production represented by

$$d_{e,q} = \left(\mu_1 p^2_{g,r} + \mu_2 p_{g,r} + \mu_3\right)/GHV, \forall q \in V_g, r \in V_e \tag{4}$$

where μ_1, μ_2 and μ_3 are coefficients depending on the gas-fired unit property (in $\mathrm{MBTU/MW^2h}$, $\mathrm{MBTU/MWh}$ and MBTU). GHV represents the gross heating value of natural gas, usually ranging from 950 to 1150 $\mathrm{BTU/ft^3}$. Observe that the coupling constraint (4) is quadratic.

III. Decomposition of E&GEP Model

A. Preliminaries on the ADMM Algorithm

Firstly, we briefly introduce the ADMM algorithm. See [12] for more details. The algorithm intends to solve problems with decomposable *linear* structure shown in (5).

$$\min_{x,y} \ f(x) + h(y), \quad \text{s.t.:} \quad Ax + By = c \tag{5}$$

After introducing Lagrange multipliers λ, and one positive penalty parameter γ, the augmented Lagrangian is

$$L(x,y,\lambda) = f(x) + h(y) + \lambda(Ax + By - c) + \gamma/2\|Ax + By - c\|^2_2 \tag{6}$$

The ADMM updates in the $(v+1)$ iteration consists of two steps of primal variables updates, (7a)–(7b), and one step of dual variable updates, (7c).

$$x^{(v+1)} = \arg\min_{x} L(x, y^{(v)}, \lambda^{(v)}), \tag{7a}$$

$$y^{(v+1)} = \arg\min_{y} L(x^{(v+1)}, y, \lambda^{(v)}), \tag{7b}$$

$$\lambda^{(v+1)} = \lambda^{(v)} + \gamma \left(Ax^{(v+1)} + By^{(v+1)} - c \right) \tag{7c}$$

The ADMM algorithm can split the original problem, (5), into two (or more) independent parts that can be solved without sharing information from other sub-problems. That is, each sub-problem can be solved in decentralized manner. Additionally, complexity of the sub-problems do not grow with each iteration.

B. Iterative ADMM Formulation for E&GEP Model

The E&GEP model can be split into two sub-problems with a decomposable strucuture for using ADMM algorithm for solving it. The natural gas value on the coupling lines need to be reconsidered into objective function for each system. Gas provided by the gas system is sold to electricity network. So gas system can make profits from the delivered gas to gas-fired generators in electricity network represented by $Pro(d_e)$. The objective function of gas system is then defined as

$$f(\mathbf{g}, d_e) = \beta C^{GEP}(\mathbf{g}) + C^{GO}(\mathbf{g}) - Pro(d_e) \tag{8}$$

Similarly, for electricity system, the following extended objective function that accounts for the cost of purchasing gas, $C(d_e)$, is obtained.

$$h(\mathbf{e}, d_e) = \beta C^{EEP}(\mathbf{e}) + C^{EO}(\mathbf{e}) + C(d_e) \tag{9}$$

In the standard ADMM algorithm, coupling constraint are usually linear in order to achieve good performance. Thus, by introducing the auxiliary variable, $\widehat{d_r}$, the quadratic coupling constraints can be transformed into a linear one.

$$\widehat{d_r} = \mu_1 p_{g,r}^2 + \mu_2 p_{g,r} + \mu_3, \tag{10}$$

the coupling constraint is reformulated in as

$$d_{e,q} \cdot GHV = \widehat{d_r}, \quad \forall q \in V_g, r \in V_e \tag{11}$$

Equation (11) becomes new coupling constraints in the E&GEP model and (10) need to be included into constraints set in the electricity network planning model.

Now, each independent system-based problem can be reformulated and solved by the conventional ADMM algorithm. But before it, a scaling parameter, S_c, is introduced to overcome some numerical issues that we may encounter in the simulation considering the large value of GHV. Thus, the gas network expansion planning model (GP) is defined as follows

$$\begin{aligned} \text{GP}(\widehat{d_r}, \lambda, \gamma) = \min_{\mathbf{g}, d_e} \Big[& f(\mathbf{g}, d_e) + \lambda(d_e \cdot GHV)/S_c \\ & + \gamma/2 \| (d_e \cdot GHV - \widehat{d_r})/S_c \|_2^2 \Big] \\ \text{s.t.: } & \text{Gas Network (2)} \end{aligned} \tag{12}$$

Simlarly, the problem for the electricity system expansion planning (EP) can be built.

$$\begin{aligned} \text{EP}(d_e, \lambda, \gamma) = \min_{\mathbf{e}, \widehat{d_r}} \Big[& h(\mathbf{e}, \widehat{d_r}) + \lambda(-\widehat{d_r})/S_c \\ & + \gamma/2 \| (d_e \cdot GHV - \widehat{d_r})/S_c \|_2^2 \Big] \\ \text{s.t.: } & \text{Electricity Network (3)} \\ & \widehat{d_r} \text{ definition (10)} \end{aligned} \tag{13}$$

The updating rule of Lagrangian multiplier λ is

$$\lambda^{(v+1)} = \lambda^{(v)} + \gamma(d_e^{(v+1)} \cdot GHV - \widehat{d_r}^{(v+1)})/S_c \tag{14}$$

The algorithm flowchart is shown in Algorithm 1.

Algorithm 1 Iterative ADMM Algorithm for the E&GEP Model

Input: Existing and new candidate topology and equipment
Output: Investment decisions and operating status

1: Initialization: $\lambda^{(1)}, d_e^{(1)}, \gamma^{(v)} = 1/\sqrt{v}, v = 1, \epsilon = 10^{-3}$
2: **while** $\zeta \geq \epsilon$ **do**
3:     **Step 1**: solve the electricity network expansion problem (13) and obtain $\widehat{d}_r^* \longleftarrow \text{EP}(d_e^{(v)}, \lambda^{(v)}, \gamma^{(v)})$
4:     update $\widehat{d_r}^{(v+1)} = \widehat{d}_r^*$
5:     **Step 2**: solve gas network expansion problem (12) and obtain $d_e^* \longleftarrow \text{GP}(\widehat{d_r}^{(v+1)}, \lambda^{(v)}, \gamma^{(v)})$
6:     update $d_e^{(v+1)} = d_e^*$
7:     **Step 3**: update the multiplier $\lambda^{(v+1)}$ based on (14).
8:     **Step 4**: update coupling mismatch and increase counter $\zeta = \max |d_e^{(v+1)} \cdot GHV - \widehat{d_r}^{(v+1)}|$
9:         $v = v + 1$
10: **end while**

IV. Case Studies

A modified version of the Belgium gas transmission network and the IEEE 14 bus system, as shown in Fig. 1, is utilized as the test system in the paper. β is set to 0.01. Data about gas pipelines, gas-fired generators are taken from [16] and [17]. Line limits in the electricity network are taken from [15]. The complete data set of the case studies used in this paper can be found in [18]. Numerical tests are performed by Julia 1.0.1 with SCIP solver on a PC with Intel(R) Core(TM) i7-6500U CPU(2.5 GHz) and 8GB memory.

Four cases are studied in the paper. In the case 1, the demand and production for gas and electricity are taken as nominal values in [14]. In the case 2, only the gas demand is increased by 30 percent compared with the case 1. In the case 3, only the

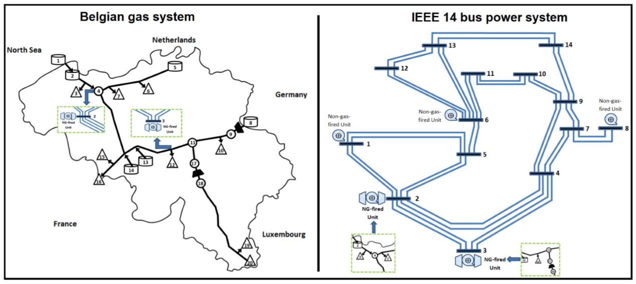

Fig. 1. Schematics of the electricity and gas network (from [15]).

electricity demand is 1.3 times the nominal value of the case 1. In case 4, the demand for gas and electricity simultaneously increase to 1.3 times with regard to the case 1.

A. E&GEP Results without Using Decomposition

The E&GEP co-optimization model (1) was solved directly without decomposition method. Table I provides the optimal solutions for 4 cases.

In Table I, N_{np} is the number of newly built pipelines, N_{nl} is the number of newly built electricity lines, and d_e contains gas demand for gas-fired generators on node 4 and node 12 of the gas network (node 2 and node 3 in electricity network, respectively), in Mm^3. TCost lies in the fifth column, representing the whole investment and operating costs form the objective objective function (in million USD). CPU time is reported in column six (in s). As the results show, when the demand and production of gas and electricity increase, several electricity lines and gas pipelines are necessary to be built and the total costs increase as well. These results are used later as benchmark .

TABLE I
E&GEP Model Results without Decomposition

Case	N_{np}	N_{nl}	d_e	TCost	CPU
1	0	0	(0.0,0.0)	8.3417	6
2	3	0	(0.0,0.0)	14.7062	9
3	0	5	(0.0,0.032353)	8.5766	8
4	3	5	(0.0,0.032353)	14.9411	50

B. E&GEP Model Results Using ADMM Decomposition

Electricity and gas systems may be unable to share private and strategic information to each other. For this case, the ADMM algorithm offers a unique advantage to address the E&GEP problem jointly. Good expansion decisions can be obtained for both electricity and gas system under the condition that only information on the coupling lines and equipment exchange between them.

To make comparison with the results in the Section IV-A, the four cases studies have been conducted using the ADDM-based algorithm presented in the Section III-B for solving the same E&GEP problem. Several simulations has been run using different scaling factors and initial values.

Tables II–V show the results when using different scaling parameter, S_c, for the 4 cases. The benchmark results (E&GEP solved without ADMM decomposition) are listed in the first row of each table.

The E&GEP model contains non-convex constraints. To the authors' best knowledge, there is no general convergence proof for ADMM application in non-convex problems. However, we did observe that the optimal results always converged to one exact point in each case when S_c is 10000 and the obtained cost is equal to the benchmark cost, while worse solutions may be obtained when S_c is 1000. It showed better performance to downscale the penalty term when Sc is set to 10000 because the coefficient in the penalty term, GHV, is 35315 $\mathrm{MBTU/Mm}^3$ in this problem. Therefore, choosing an appropriate scaling parameter for the coupling constraints is of significance to achieve good results when applying ADMM.

TABLE II
E&GEP Model Solution of Case 1 from ADMM Algorithm

S_c	$d_e^{(1)}$	$\lambda^{(1)}$	N_{np}	N_{nl}	d_e	TCost	CPU
-	-	-	0	0	(0.0,0.0)	8.3417	6
1000	(0.0,0.0)	(1,1)	0	0	(0.0,0.0)	8.3417	3
1000	(0.0,0.032353)	(1,1)	0	0	(0.0,0.00028)	8.3417	62
1000	(0.05,0.0)	(1,1)	0	1	(0.04913,0.0)	8.3926	12
1000	(0.05,0.05)	(1,1)	0	0	(0.0364,0.0355)	8.3550	10
10000	(0.0,0.032353)	(1,1)	0	0	(0.0,0.0)	8.3417	24
10000	(0.0,0.0)	(1,1)	0	0	(0.0,0.0)	8.3417	24
10000	(0.05,0.0)	(1,1)	0	0	(0.0,0.0)	8.3417	25
10000	(0.05,0.05)	(1,1)	0	0	(0.0,0.0)	8.3417	26

TABLE III
E&GEP Model Solution of Case 2 from ADMM Algorithm

S_c	$d_e^{(1)}$	$\lambda^{(1)}$	N_{np}	N_{nl}	d_e	TCost	CPU
-	-	-	3	0	(0.0,0.0)	14.7062	9
1000	(0.0,0.0)	(1,1)	3	0	(0.0,0.0)	14.7062	9
1000	(0.0,0.032353)	(1,1)	3	0	(0.0,0.0)	14.7062	251
1000	(0.0,0.05)	(1,1)	3	0	(0.0,0.0)	14.7062	327
1000	(0.05,0.0)	(1,1)	3	1	(0.04914,0.0)	14.7607	25
1000	(0.05,0.05)	(1,1)	3	0	(0.0380,0.0369)	14.720	13
10000	(0.0,0.0)	(1,1)	3	0	(0.0,0.0)	14.7062	57
10000	(0.0,0.032353)	(1,1)	3	0	(0.0,0.0)	14.7062	64
10000	(0.0,0.05)	(1,1)	3	0	(0.0,0.0)	14.7062	53
10000	(0.05,0.0)	(1,1)	3	0	(0.0,0.0)	14.7062	54
10000	(0.05,0.05)	(1,1)	3	0	(0.0,0.0)	14.7062	51

TABLE IV
E&GEP Model Solution of Case 3 from ADMM Algorithm

Sc	$d_e^{(1)}$	$\lambda^{(1)}$	N_{np}	N_{nl}	d_e	TCost	CPU
-	-	-	0	5	(0,0.032353)	8.5766	8
10	(0,0.032353)	(1,1)	0	5	(0,0.032353)	8.5766	15
100	(0,0.032353)	(1,1)	0	5	(0,0.032353)	8.5766	44
1000	(0,0.032353)	(1,1)	0	5	(0,0.032353)	8.5766	12
1000	(0,0)	(1,1)	0	6	(0.0111,0.0111)	8.6197	49
1000	(0,0.05)	(1,1)	0	5	(0,0.04881)	8.5795	17
1000	(0.05,0)	(1,1)	0	7	(0.04742,0)	8.6698	61
1000	(0.05,0.05)	(1,1)	0	6	(0.04949,0.04960)	8.6345	24
10000	(0,0.032353)	(1,1)	0	5	(0,0.032353)	8.5766	71
10000	(0,0)	(1,1)	0	5	(0,0.032353)	8.5766	130
10000	(0,0.05)	(1,1)	0	5	(0,0.032353)	8.5765	68
10000	(0.05,0)	(1,1)	0	5	(0,0.032353)	8.5766	115
10000	(0.05,0.05)	(1,1)	0	5	(0,0.032353)	8.5765	59

TABLE V
E&GEP MODEL SOLUTION OF CASE 4 FROM ADMM ALGORITHM

S_c	$d_e^{(1)}$	$\lambda^{(1)}$	N_{np}	N_{nl}	d_e	TCost	CPU
-	-	-	3	5	(0,0.032353)	14.9411	50
1000	(0,0.032353)	(1,1)	3	5	(0,0.032353)	14.9411	19
1000	(0,0)	(1,1)	3	6	(0.0111,0.0111)	14.9841	96
1000	(0,0.05)	(1,1)	3	5	(0,0.04878)	14.9440	31
1000	(0.05,0)	(1,1)	3	7	(0.04879,0)	15.03	55
1000	(0.05,0.05)	(1,1)	3	5	(0.0473,0.0478)	14.998	89
10000	(0,0.032353)	(1,1)	3	5	(0,0.032353)	14.9411	112
10000	(0,0)	(1,1)	3	5	(0,0.032353)	14.9411	72
10000	(0,0.05)	(1,1)	3	5	(0,0.032353)	14.9411	141
10000	(0.05,0)	(1,1)	3	5	(0,0.032353)	14.9411	66
10000	(0.05,0.05)	(1,1)	3	5	(0,0.032353)	14.9411	151

Fig. 2 shows the convergence process in the case 4 when S_c, $d_e^{(1)}$ and $\lambda^{(1)}$ are initialized to 10000, (0.05,0.05) and (1,1), respectively. The obtained results get closer to the optimal solution while the number of iterations increase until it ends. It is observed that after 7 iterations the error is almost null. On the other hand, during the process of experiment, we tried to set $\lambda^{(1)}$ to (10,10) and the scaling parameter to 10000. We noticed that λ decreased only in a quite small step within each iteration, which needed much longer time to converge than when $\lambda^{(1)}$ is set to be (1,1).

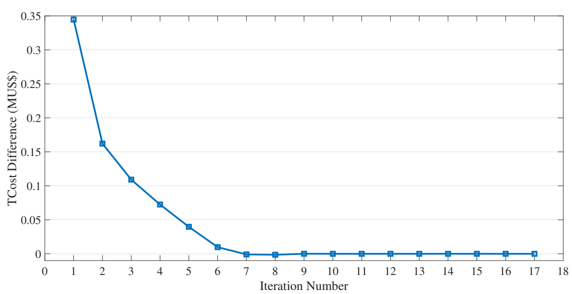

Fig. 2. Convergence process after ADMM application.

V. CONCLUSIONS

This paper proposes a coupled electricity and gas expansion planning model. The ADMM-based algorithm has been proposed to overcome data privacy between both systems. In addition, we have derived new reformulation for the non-linear coupling gas-electricity constraint to transform into a linear one. In this way the E&GEP problem is decomposed to two sub-models and solved in an iterative way. The first advantage is the size of problem is cut into half, which makes it easier for the solver to get the solution. Since there is information privacy between the gas industry and electricity industry, there is no need for them to share all the information they have but only the coupling gas and electricity's supply and consumption.

The algorithm has been tested on several simulation experiments for both cases, with and without decomposition. The results show good convergence under very general conditions for different initial starting points and scaling parameters. Despite the good performance of the ADMM showed for this problem in the experiments, there is no convergence guarantee of the ADMM algorithm for non-convex problems. Our future research will focus on finding a convexified reformulation for the E&GEP problem that implicitly grants convergence.

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