

# Лекция 9: Задачи множественного доступа и их связь с задачей сжатия измерений.

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1 Problem statement

2 3GPP proposals

3 System model

4 State-of-the-art

5 Polar-coded IRSAs

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Main features:

- ▶ huge amount of autonomous devices (hundreds of thousands) connected to a single base station
- ▶ short data packets
- ▶ different types of traffic according to access scenarios
- ▶ require extremely low energy consumption

**Grant-free transmission** is more desirable for 5G mMTC as the main goal is to reduce hardware complexity and improve energy efficiency.

# Outline

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- ▶ Multi-user shared access (MUSA) (e.g., R1-162226)
- ▶ Resource spread multiple access (RSMA) (e.g., R1-163510)
- ▶ Sparse code multiple access (SCMA) (e.g., R1-162153)
- ▶ Pattern defined multiple access (PDMA) (e.g., R1-163383)
- ▶ Non-orthogonal coded multiple access (NCMA) (e.g., R1-162517)
- ▶ Low code rate spreading (e.g., R1-162385)
- ▶ Frequency domain spreading (e.g., R1-162385)
- ▶ Non-orthogonal multiple access (NOMA) (e.g., R1-163111)

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- ▶  $\|X(\omega)\|_2^2 \leq nP$ , which means a natural power constraint.

Decoding is done up to permutation of messages. We only require the decoder to output a set  $\mathcal{L}(Y) = (X_1, X_2, \dots, X_K) \in [M]^K$ .

Per User Probability of Error

$$p_e = \max_{|(s_1, s_2, \dots, s_{K_{\text{tot}}})| = K} \frac{1}{K} \sum_{i=1}^{K_{\text{tot}}} s_i \Pr(X_i \notin \mathcal{L}(Y)).$$



Y. Polyanskiy.

A perspective on massive random-access

in Proc. IEEE Int. Symp. on Inf. Theory (ISIT), 2017, pp. 2523–2527, 2017.

$$Y = \sum_{i=1}^{K_{\text{tot}}} s_i X_i + Z$$

- ▶  $s_i$  – activity indicator
- ▶  $Z \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$

A user transmits  $k$  bits by means of  $n$  channel uses, then

$$E_b/N_0 = \frac{nP}{2k}$$

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-  Y. Polyanskiy.  
A perspective on massive random-access  
in Proc. IEEE Int. Symp. on Inf. Theory (ISIT), pp. 2523–2527, 2017.
-  I. Zadik, Y. Polyanskiy and C. Thrampoulidis.  
Improved bounds on Gaussian MAC and sparse regression via Gaussian inequalities  
in Proc. IEEE Int. Symp. on Inf. Theory (ISIT), pp. 430–434, 2019.

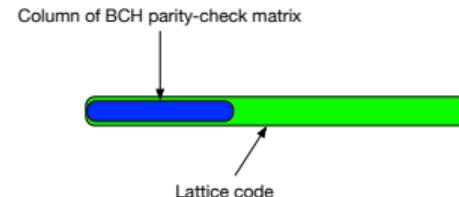
## What about low-complexity coding schemes?

$$\mathbf{y} = \mathbf{As} + \mathbf{z},$$

where

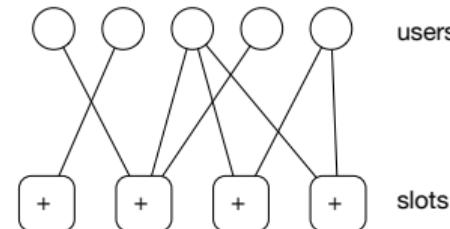
- ▶  $\mathbf{y}$  – channel output
- ▶  $\mathbf{A}$  – codebook of size  $n \times 2^k$
- ▶  $\mathbf{s}$  – activity vector,  $\mathbf{s} \in \{0, 1\}^{2^k}$
- ▶  $\mathbf{z}$  – Gaussian noise

The complexity is too big!



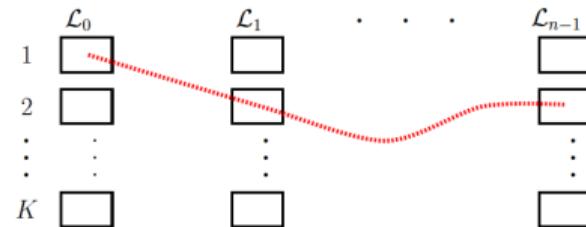
O. Ordentlich and Y. Polyanskiy.

Low complexity schemes for the random access gaussian channel  
in Proc. IEEE Int. Symp. on Inf. Theory (ISIT), pp. 2528–2532, 2017.



- ☞ A. Vem, K. R. Narayanan, J. Cheng, and J.-F. Chamberland.  
Low complexity schemes for the random access gaussian channel  
in Proc. IEEE Information Theory Workshop (ITW), pp. 1–5, 2017.
- ☞ A. Glebov, N. Matveev, K. Andreev, A. Frolov, A. Turlikov.  
Achievability Bounds for T-Fold Irregular Repetition Slotted ALOHA Scheme in the Gaussian MAC  
in Proc. IEEE Wireless Communications and Networking Conference (WCNC), pp. 1–5, 2019.

# Compressive Sensing approach

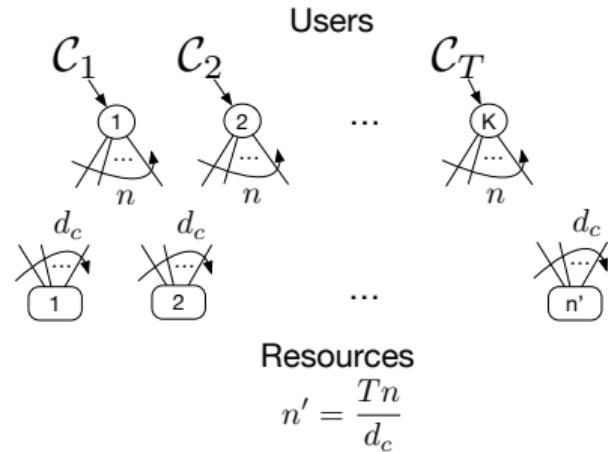


- ☞ V. K. Amalladinne, A. Vem, D. K. Soma, K. R. Narayanan and J. Chamberland  
A Coupled Compressive Sensing Scheme for Unsourced Multiple Access  
IEEE International Conference on Acoustics, Speech and Signal Processing  
(ICASSP), Calgary, AB, 2018, pp. 6628-6632.
- ☞ R. Calderbank and A. Thompson,  
CHIRRUP: a practical algorithm for unsourced multiple access  
arXiv preprint arXiv:1811.00879, 2018.



A. Fengler, P. Jung, and G. Caire,  
SPARCs for Unsourced Random Access,  
arXiv preprint arXiv:1901.06234, 2019.

# Joint decoding graph approach



A. Pradhan, V. Amalladinne, A. Vem, K. R. Narayanan, and J.-F. Chamberland  
A joint graph based coding scheme for the unsourced random access gaussian channel  
arXiv preprint arXiv:1906.05410, 2019.

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Polar codes in combination with Tal–Vardy list decoder are extremely effective for short code lengths and low code rates.



E. Arikan

Channel polarization: A method for constructing capacity-achieving codes for symmetric binary-input memoryless channels

IEEE Transactions on Information Theory, vol. 55, no. 7, pp. 3051–3073, 2009.



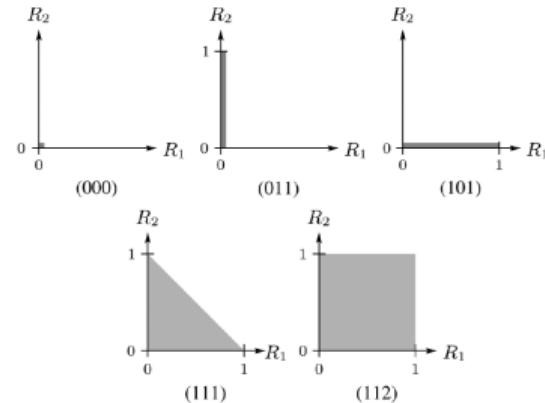
I. Tal and A. Vardy,

List decoding of polar codes

IEEE Transactions on Information Theory, vol. 61, no. 5, pp.2213–2226, 2015.

Why not to use them in our scenario?

# GMAC polarization



- ☞ E. Sasoglu, E. Telatar, and E. M. Yeh  
Polar codes for the two-user multiple-access channel  
IEEE Transactions on Information Theory, vol. 59, no. 10, pp. 6583–6592, 2013
- ☞ E. Abbe and E. Telatar,  
Polar codes for them-user multiple access channel  
IEEE Transactions on Information Theory, vol. 58, no. 8, pp. 5437–5448, 2012



E. Arikan

Polar coding for the slepian-wolf problem based on monotone chain rules

IEEE International Symposium on Information Theory Proceedings, Cambridge, MA, 2012, pp. 566–570, 2012.



S. Onay,

Successive cancellation decoding of polarcodes for the two-user binary-input MAC

IEEE International Symposium on Information Theory, pp. 1122–1126, 2013



H. Mahdavifar, M. El-Khamy, J. Lee, and I. Kang,

Achieving the uniform rate region of general multipleaccess channels by polar coding

IEEE Transactions on Communications, vol. 64, no. 2, pp. 467–478, Feb 2016

How to construct and decode polar codes for GMAC in finite length regime?

Consider the Arikan's kernel and the corresponding *polar transform* of size  $N = 2^n$

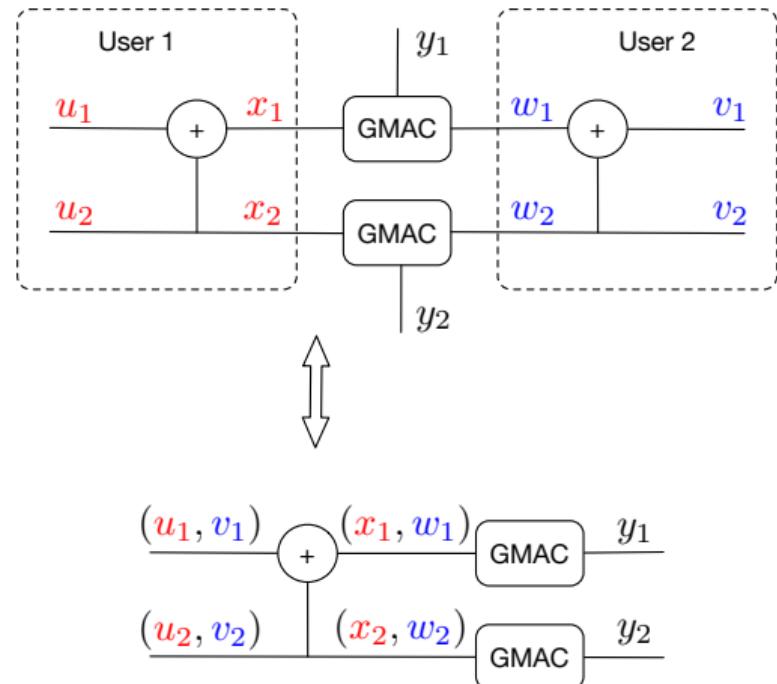
$$G_2 \triangleq \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \quad G_N \triangleq B_N G_2^{\otimes n},$$

where  $\otimes$  is the Kronecker power and  $B_N$  is called a *shuffle reverse* operator.

Denote the set of frozen positions by  $F$ ,  $|F| = N - k$ . By  $\mathbf{u}_F$  we denoted the projection of the vector  $\mathbf{u}$  to positions in  $F$ . For now, we can define a *polar coset code*  $\mathcal{C}$  as follows

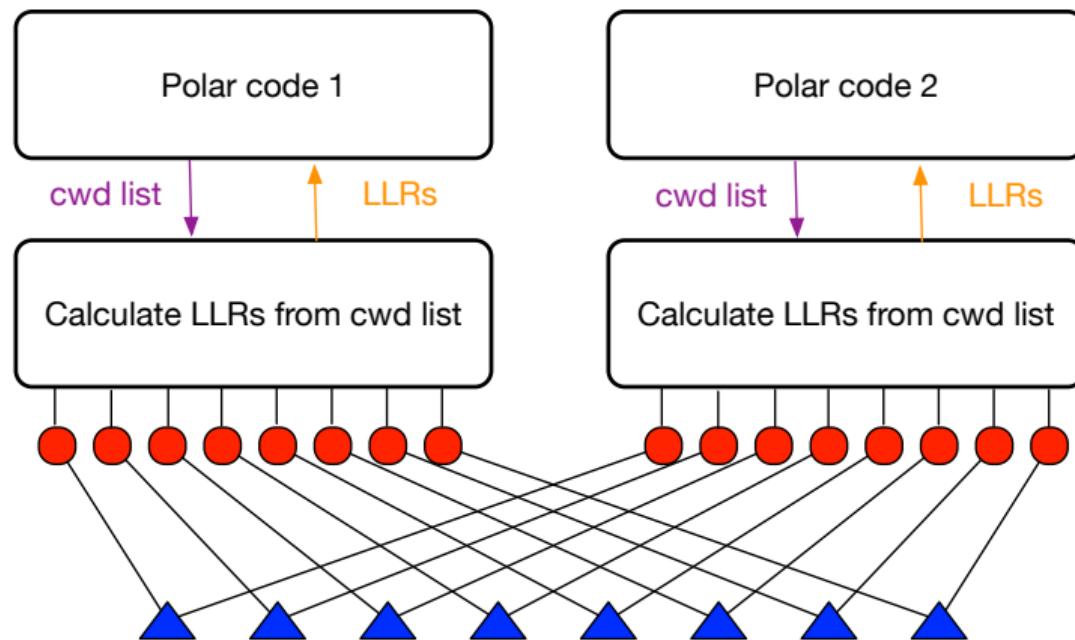
$$\mathcal{C}(N, k, F, \mathbf{f}) = \left\{ \mathbf{c} = \mathbf{u} G_N \mid \mathbf{u} \in \{0, 1\}^N, \quad \mathbf{u}_F = \mathbf{f} \right\}.$$

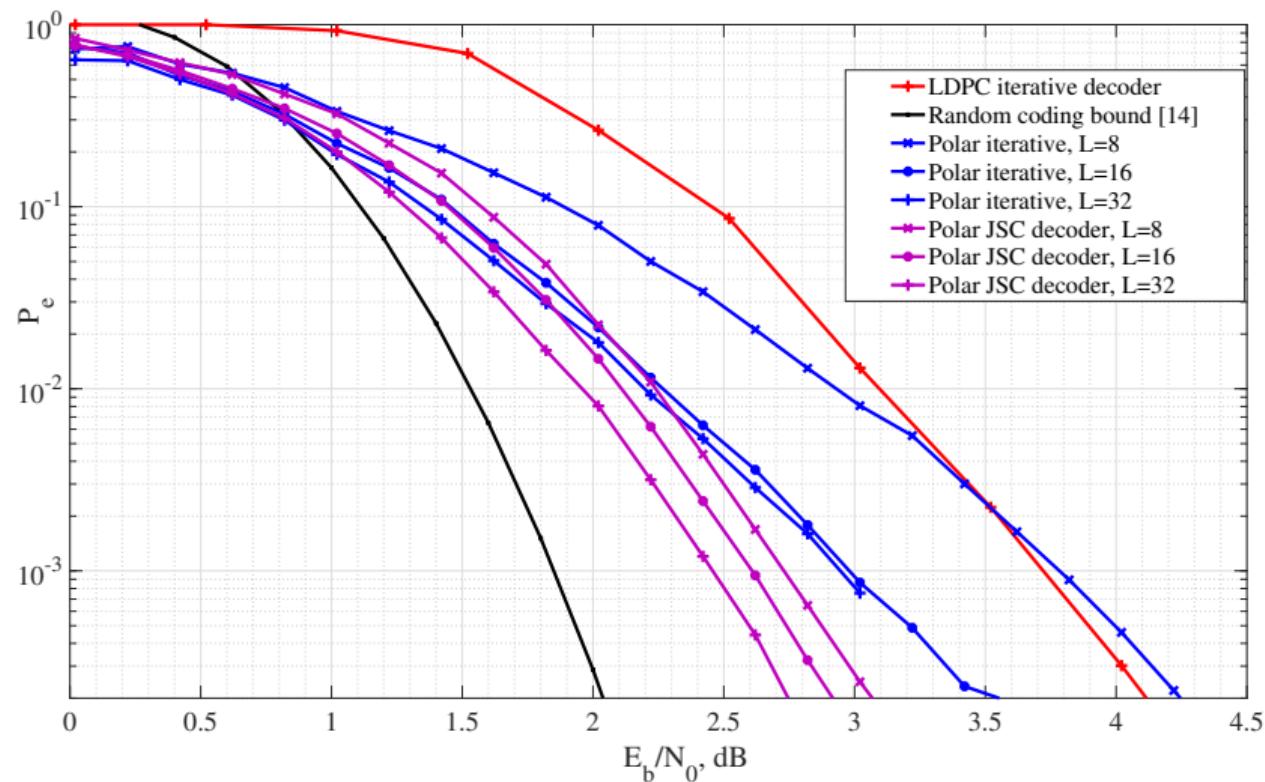
# Joint Successive Cancellation Decoding

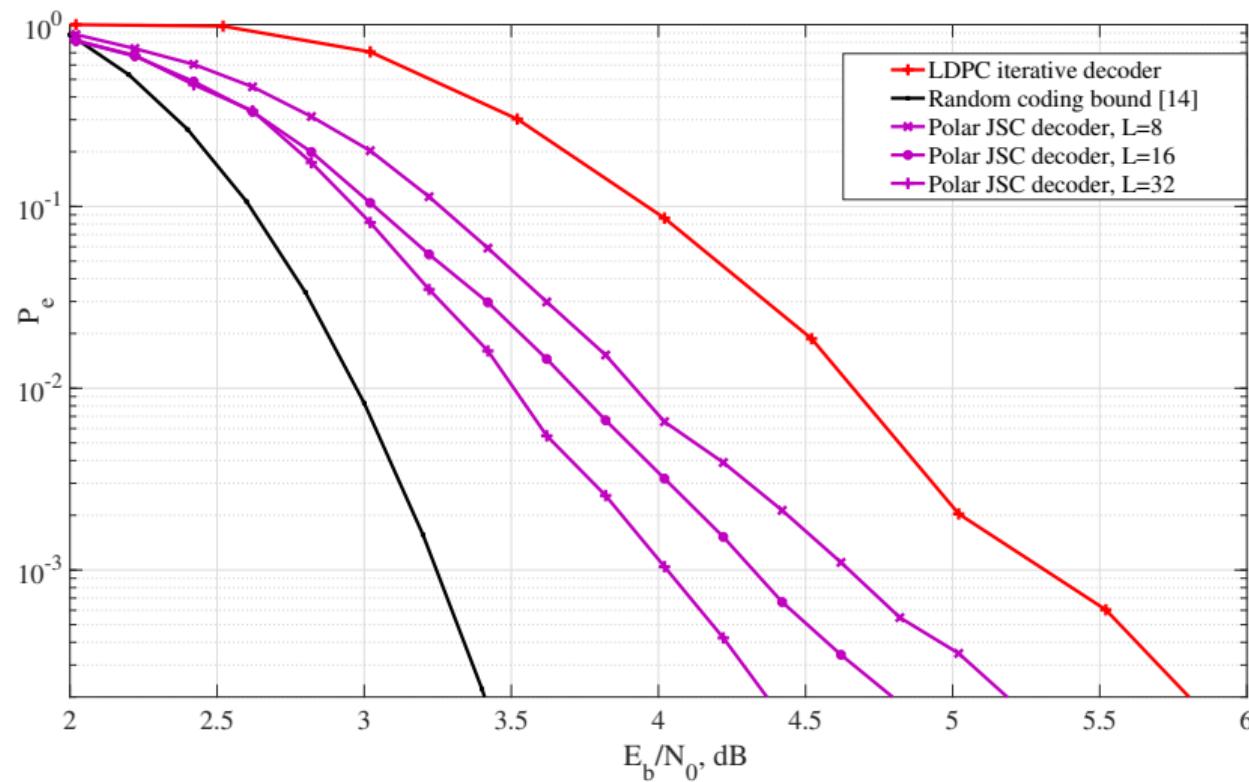


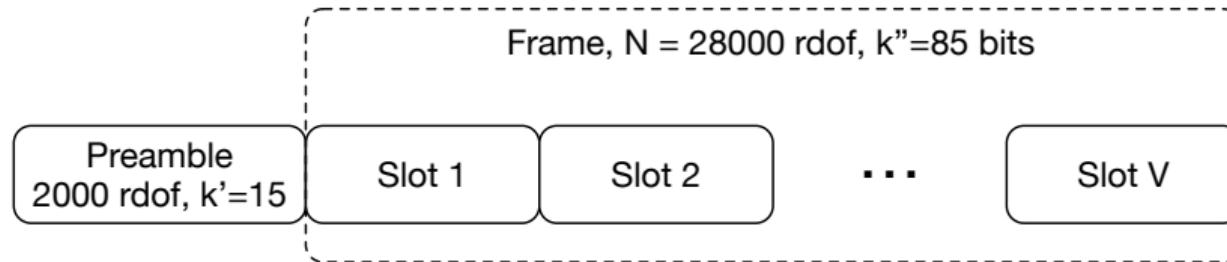
Representation as a polar code over  $\mathbb{Z}_2^K$  for  $K = 2$ .

# Iterative decoding

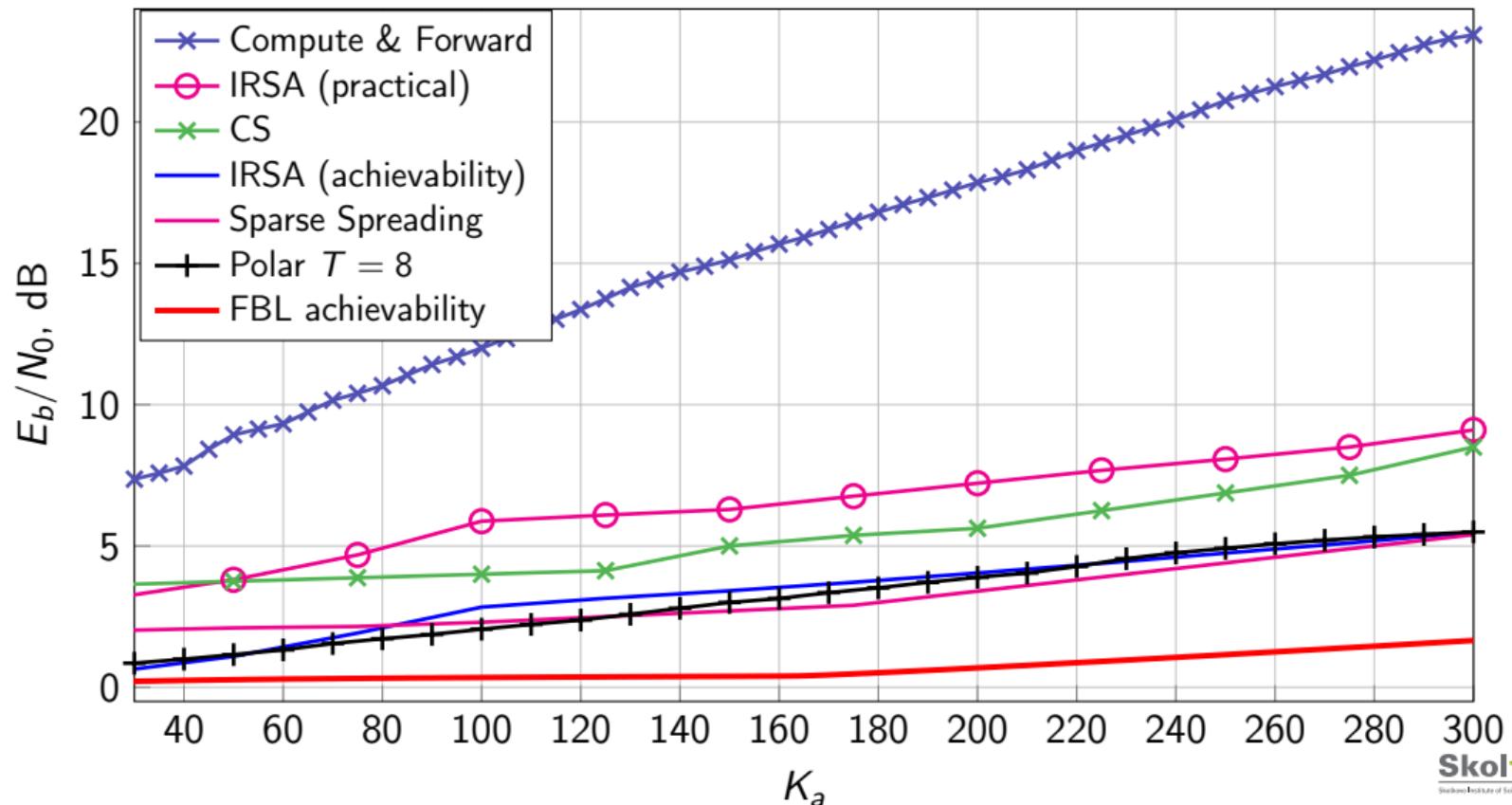








# Results for Gaussian MAC



- ▶ We compared two possible decoding techniques: joint successive cancellation algorithm and joint iterative algorithm.
- ▶ In order to optimize the codes (choose frozen bits) we proposed a specialized and efficient design algorithm.
- ▶ We investigated the performance of the resulting scheme in the Gaussian multiple access channel (GMAC) by means of simulations.
- ▶ Our scheme outperforms LDPC based solution by approximately 1 dB and is closer to the achievability bound for GMAC.

- ▶ consider fading channels.
- ▶ another important factor is frame-synchronization which we have assumed. It is interesting to see how the performance is affected when frame synchrony is lost.
- ▶ consider MIMO scenario with the receiver having multiple antennas.

# $T$ -fold irregular repetition slotted ALOHA (IRSA)

- ▶ E. Casini, R. De Gaudenzi, and O. del Rio Herrero, Contention resolution diversity slotted ALOHA (CRDSA): An enhanced random access scheme for satellite access packet networks, IEEE Trans. Wireless Commun., vol. 6, no. 4, pp. 1408–1419, Apr. 2007.
- ▶ G. Liva, Graph-based analysis and optimization of contention resolution diversity slotted aloha, IEEE Transactions on Communications, vol. 59, no. 2, pp. 477–487, February 2011.
- ▶ K. R. Narayanan and H. D. Pfister, Iterative collision resolution for slotted aloha: An optimal uncoordinated transmission policy, in Proc. 7th International Symposium on Turbo Codes and Iterative Information Processing (ISTC), Aug 2012, pp. 136–139.
- ▶ M. Ghanbarinejad and C. Schlegel, “Irregular repetition slotted aloha with multiuser detection,” in Proc. 10th Annual Conference on Wireless On-demand Network Systems and Services (WONS), March 2013, pp. 201–205.
- ▶ C. Stefanovic, E. Paolini, and G. Liva, Asymptotic performance of coded slotted aloha with multi-packet reception,”IEEE Communications Letters, vol. 22, no. 1,pp. 105–108, Jan 2018.
- ▶ F. Clazzer, E. Paolini, I. Mambelli, and C. Stefanovic, Irregular repetition slotted aloha over the rayleigh block fading channel with capture, in Proc. 2017 IEEE International Conference on Communications (ICC), May 2017, pp. 1–6.

Thank you for your attention!