Camera Pose Estimation from Line and Point Correspondences

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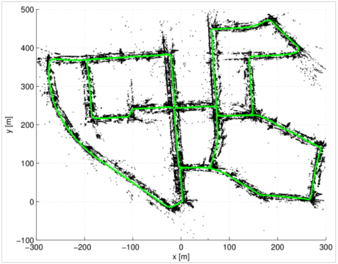
Talk Outline

Some Applications of Multiple View Geometry

Structure from Motion	Visual SLAM
Bundler (2006)	MonoSLAM (2007)
Visual SFM (2013)	PTAM (2007)
Theia (2015)	LSD-SLAM (2014)
COLMAP (2016)	ORB-SLAM (2014)

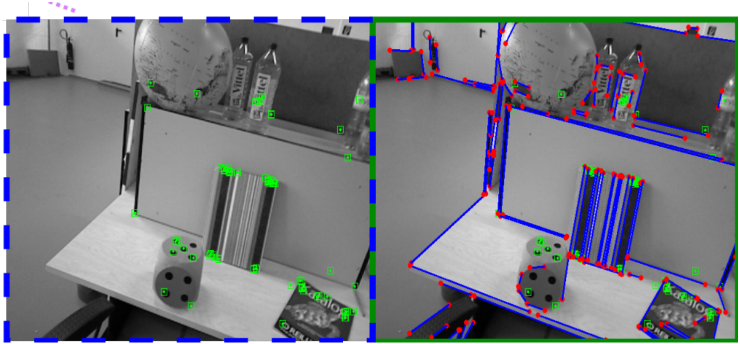
Bundler (N. Snavely et al., 2006)



ORB-SLAM (Mur et al., 2015)

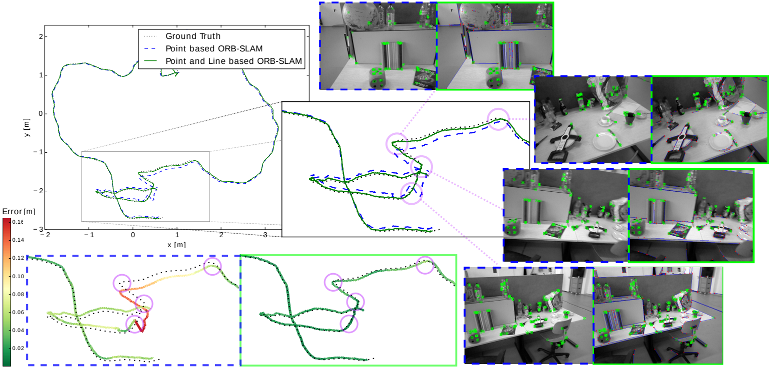
Features in Multiple View Geometry

- Points the **only** widely used in Visual SLAM and SfM features
- But we need more...



(TUM-RGBD dataset, image kindly provided by A. Pumarola, IRI-UPC)

Lines Meet ORB-SLAM



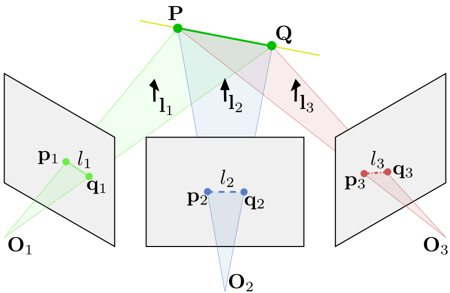
(TUM-RGBD dataset, image kindly provided by A. Pumarola, IRI-UPC)

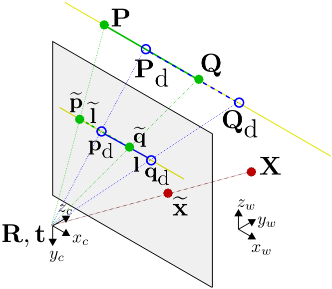
Notation

$\mathbf{U}$ - point of $\mathbb{R}^3$
$\mathbf{u}$ - point of $\mathbb{R}^2$
$\mathtt{R}$ - matrix

Some Geometric Tasks in Incremental SfM

- *Relative pose.* Having two or three 2D images, find their relative location (position+orientation) in space
- *Absolute pose.* Having 3D model and 2D image, estimate camera location w.r.t. the model

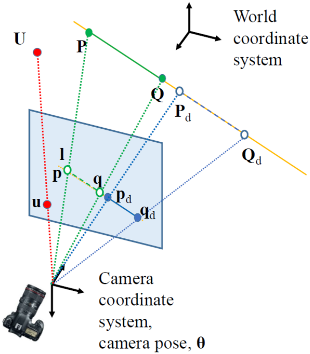




Line Matching Difficulties

- Blue - detected
- Green - reprojected
- White - manually marked model contours

Perspective-n-Point+Lines

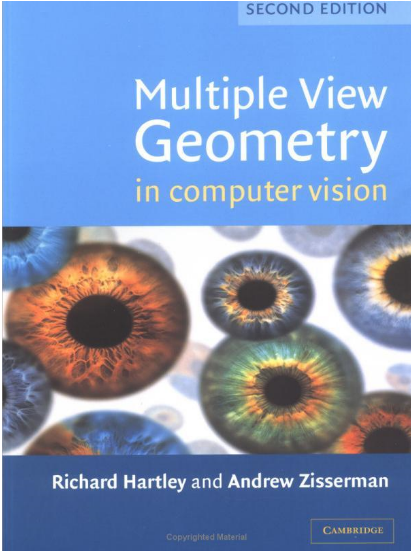


- Points: 3D $\mathbf{U}$ and 2D $\mathbf{u}$
- Line segments: 3D $\mathbf{P}, \mathbf{Q}$ and 2D $\mathbf{p}, \mathbf{q}$
- Detected line segment $\mathbf{p}_\mathrm{d}, \mathbf{q}_\mathrm{d}$, its reprojection onto 3D line $\mathbf{P}_\mathrm{d}, \mathbf{Q}_\mathrm{d}$
- model has n_p points, n_l lines
- $\boldsymbol{\theta}$ - camera pose parameters encoding $\mathtt{R}, \mathbf{t}$
- normalized camera (unit focal, zero center shift)

Perspective-n-Point+Lines

We can construct 3D model using 3 frames with SIFT point descriptors and SMLSD line descriptors (we need observation redundancy to filter outliers)
Dataset: NYU2.

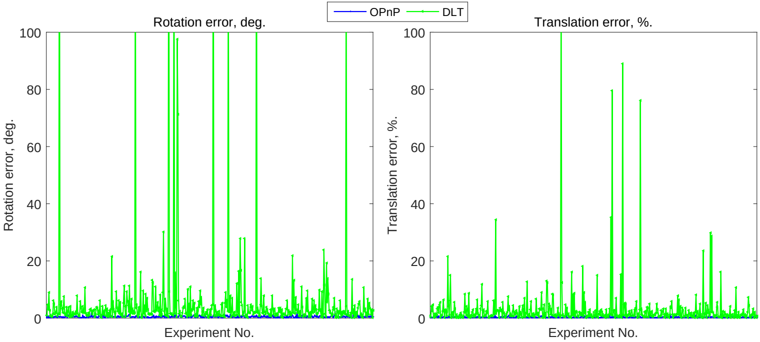
Motivating Example



If we wish to solve PnP, we open a book...
It offers Direct Linear Transform algorithm. Let's try?

Motivating Example (2). DLT vs OPnP

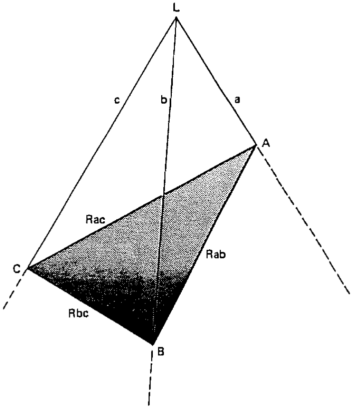
Numerical experiment: we generate 6 points in a box $[-2,2] \times [-2,2] \times [4,8]$ in front of the camera, project them with additive gaussian noise with std.dev. 1 pixel onto usual 60°-wide camera, generate random $\mathtt{R}$, $\mathbf{t}$, give a rotated by $\mathtt{R}^{-1}$ and shifted by $-\mathtt{R}^{-1}\mathbf{t}$ model to the methods, willing to get $\mathtt{R}, \mathbf{t}$.



Problem History

- XIX- beginning of XX cent. - first projective geometry results, numerical algorithms for photogrammetry
- 1960s polynomial system solving developed (Buchberger, under Groebner's supervision)
- 1970s-1980s - first PC algorithms, projective resonstruction methods (DLT)
- 1990s-2000s - minimal problem methods (P3P, P2P1L, P4Pf, etc) using Groebner bases
- 1990s - locally converging iterative algorithms for PnP
- 2000s - efficient robust algorithms for hundreds of points
- 2010s - PnP using Groebner bases

P3P Algorithm



(Fischler, Bolles, 1981)
3Cosine theorem for triangles with vertex **L**, we get 3 quadratic equations w.r.t. a, b, c, reducable to 4th order one variable equation.
At most, 4 solutions.

Direct Linear Transformation (Abdel-Aziz, Y. et al., 1971)

Homogeneous camera coordinates: $\tilde{\mathbf{x}} = \begin{pmatrix} \mathbf{x} \\ 1 \end{pmatrix}$.

Perspective projection with matrix $\mathtt{P}$:

$$\lambda \tilde{\mathbf{x}} = \mathtt{P} \begin{pmatrix} \mathbf{X} \\ 1 \end{pmatrix} \tag{1}$$

Get rid of λ:

$$\tilde{\mathbf{x}} \times \left(\mathtt{P} \begin{pmatrix} \mathbf{X} \\ 1 \end{pmatrix} \right) = 0 \tag{2}$$

For $n \geq 6$, we find $\mathtt{P}$.

Euclidean reconstruction with DLT

Having $\mathtt{P}$, $\mathtt{R}, \mathtt{t}$ - ?

$$\bar{\mathbf{X}} = \frac{1}{n}\sum_i \mathbf{X}_i$$

$$\mathtt{C} = \frac{1}{n}\sum_i (\mathbf{X}_i - \bar{\mathbf{X}})(\mathtt{P}\begin{pmatrix} \mathbf{X}_i - \bar{\mathbf{X}} \\ 1 \end{pmatrix})^T.$$

Orthogonal Procrustes

$$[\mathtt{U}, \mathtt{S}, \mathtt{V}] = SVD(\mathtt{C}) \quad \Longrightarrow \quad \mathtt{R} = \mathtt{U}\mathtt{V}^T.$$

$$\sum_i \|\mathtt{R}\mathbf{X}_i + \mathbf{t} - s\mathtt{P}\begin{pmatrix} \mathbf{X}_i \\ 1 \end{pmatrix}\|^2 \to \min_{s,\mathbf{t}}.$$

General scheme of PnP method

Having $\pi(\boldsymbol{\theta}, \mathbf{X})$ - projection function acting on 3D model point $\mathbf{X}$ outputting homogeneous camera projection coordinates $\tilde{\mathbf{x}}$:

$$\lambda_i \tilde{\mathbf{x}} = \pi(\boldsymbol{\theta}, \mathbf{X}) + \lambda_i \tilde{\xi}_i, \quad \pi(\boldsymbol{\theta}, \mathbf{X}) = \begin{pmatrix} \pi^{(1)}(\boldsymbol{\theta}, \mathbf{X}) \\ \pi^{(2)}(\boldsymbol{\theta}, \mathbf{X}) \\ \pi^{(3)}(\boldsymbol{\theta}, \mathbf{X}) \end{pmatrix}, \tag{3}$$

where $\tilde{\xi}_i = \begin{pmatrix} \xi_i \\ 1 \end{pmatrix}$, ξ_i - detection noise.

Example

$$\pi(\boldsymbol{\theta}, \mathbf{X}) = \mathtt{R}(\mathbf{q})\mathbf{X} + \mathbf{t}, \quad q = (a, b, c, d), \quad \|q\| = 1$$

$$\mathtt{R}(\mathbf{q}) = \begin{pmatrix} a^2 + b^2 - c^2 - d^2 & 2bc - 2ad & 2bd + 2ac \\ 2bc + 2ad & a^2 - b^2 + c^2 - d^2 & 2cd - 2ab \\ 2bd - 2ac & 2cd + 2ab & a^2 - b^2 - c^2 + d^2 \end{pmatrix}.$$

General scheme of PnP method (2)

$$\lambda_i \tilde{\mathbf{x}} = \pi(\boldsymbol{\theta}, \mathbf{X}) + \lambda_i \tilde{\xi}_i$$

From eq. 3 express λ_i and substitute into 1,2:

$$\pi^{(3)}(\boldsymbol{\theta}, \mathbf{X})\mathbf{x} = \pi^{(1,2)}(\boldsymbol{\theta}, \mathbf{X}). \tag{4}$$

Get a system of $2n$ eqs. (4) and solve it in least squares sense:

$$E_p = \sum_i \|\pi^{(3)}(\boldsymbol{\theta}, \mathbf{X})\mathbf{x} - \pi^{(1,2)}(\boldsymbol{\theta}, \mathbf{X})\|^2 \to \min. \tag{5}$$

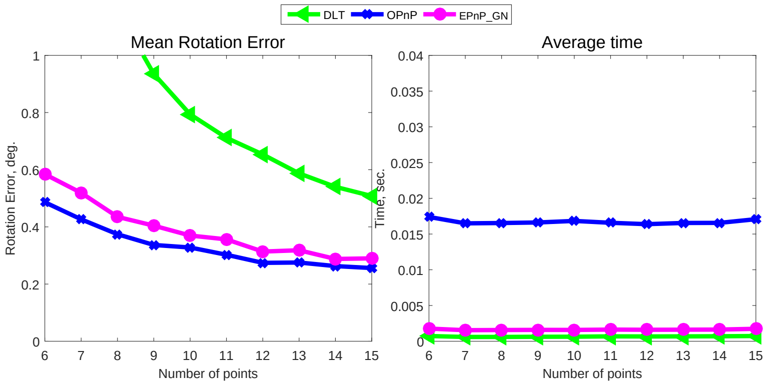
OPnP vs EPnP

$$E_p = \sum_i \|\pi^{(3)}(\boldsymbol{\theta}, \mathbf{X})\mathsf{x} - \pi^{(1,2)}(\boldsymbol{\theta}, \mathbf{X})\|^2 \to \min .$$

$$\nabla_\theta E_p = 0.$$

OPnP - $\pi(\boldsymbol{\theta}, \mathsf{x})$ - polynomial, $dim(\boldsymbol{\theta}) = 4$, solve polynomial equations
EPnP - $\pi(\boldsymbol{\theta}, \mathsf{x})$ - linear, $dim(\boldsymbol{\theta}) = 12$, but there are quadratic constraints on the components of $\boldsymbol{\theta}$. Relinearization.

Comparison of OPnP and EPnP



Generalization to PnPL

Line equation from the detected segment endpoints:

$$\hat{\mathbf{l}}^i = \tilde{\mathbf{p}}^i_{\mathrm{d}} \times \tilde{\mathbf{q}}^i_{\mathrm{d}}, \quad \mathbf{l}^i = \frac{\hat{\mathbf{l}}^i}{|\hat{\mathbf{l}}^i|} \quad \in \mathbb{R}^3. \tag{6}$$

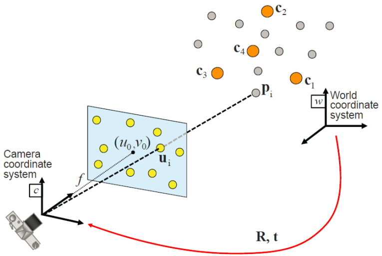
Algebraic point-to-line distance:

$$\mathrm{E}_{\mathrm{pl}}(\boldsymbol{\theta}, \mathbf{P}^i, \mathbf{l}^i) = (\mathbf{l}^i)^\top \pi(\boldsymbol{\theta}, \mathbf{P}^i), \tag{7}$$

Algebraic segment-to-line distance:

$$\mathrm{E}_{\mathrm{l}}(\boldsymbol{\theta}, \mathbf{P}^i, \mathbf{Q}^i, \mathbf{l}^i) = \mathrm{E}^2_{\mathrm{pl}}(\boldsymbol{\theta}, \mathbf{P}^i, \mathbf{l}^i) + \mathrm{E}^2_{\mathrm{pl}}(\boldsymbol{\theta}, \mathbf{Q}^i, \mathbf{l}^i). \tag{8}$$

Efficient PnP



(Lepetit, Moreno-Noguer, Fua, 2007; 2009)
First $O(n)$ algotithm for PnP.

Choose 4 control points $\mathbf{C}_i$, not in one plane.

$$\mathbf{P}_i = a_{i,1}\mathbf{C}_1 + a_{i,2}\mathbf{C}_2 + a_{i,3}\mathbf{C}_3 + a_{i,4}\mathbf{C}_4$$

$$\forall \mathtt{R}, \mathbf{t}: \quad \mathtt{R}\mathbf{P}_i + \mathbf{t} = \sum_j a_{i,j}(\mathtt{R}\mathbf{C}_j + \mathbf{t})$$

$a_{i,j}$ do not change under rotation and translation.

Efficient PnP (2)

$$\pi_{\mathrm{EPnP}}(\boldsymbol{\theta}, \mathbf{X}_i) = \sum_{j=1}^{4} a_{i,j}\mathbf{C}_j \tag{9}$$

We get 2 equations w.r.t. $\boldsymbol{\mu} = (\mathbf{C}_{c,1}^T, \mathbf{C}_{c,2}^T, \mathbf{C}_{c,3}^T, \mathbf{C}_{c,4}^T)$ for a single 3D-2D match, пo using all correspondences we form a linear system:

$$\mathbf{M}\boldsymbol{\mu} = \mathbf{0}, \quad \mathbf{M} \in \mathbb{R}^{2n\times 12} \tag{10}$$

Without noise ξ_i in point detections, $\mathbf{M}$ has a null space of dimension 1. But, in real life, we need to seek for a solution in a linear subspace of the singular vectors $\mathbf{v}_1, \ldots, \mathbf{v}_N$ $\mathbf{M}$, corresponding to $N = 1, 2, 3, 4$ smallest singular values of $\mathbf{M}$.

Efficient PnP (3)

So, $\boldsymbol{\mu}$ can be represented as

$$\boldsymbol{\mu} = \sum_{i=1}^{N} \beta_i \mathbf{v}_i \tag{11}$$

To find a unique solution we use invariance of distance between points under rotation and translation:

$$\|\mathbf{C}_i - \mathbf{C}_j\|^2 = r_{ij}^2, \quad i, j = 1, \ldots, 4, i \neq j. \tag{12}$$

Substitute (11) in (12), get quadratic system of 6 equations w.r.t. $\beta_1, \ldots, \beta_N$.
We solve it using relinearization, defining $\beta_i \beta_j$ as new unknowns.

Efficient PnP (4): Relinearization

Denote $\gamma_k = \beta_i \beta_j$, compose $\boldsymbol{\gamma} = (\gamma_1, \ldots, \gamma_{N+N(N-1)/2})^T$, $\mathbf{w} = (r_{12}, \ldots, r_{34})^T$ and get a system:

$$\Gamma \boldsymbol{\gamma} = \mathbf{w}. \tag{13}$$

Problem: when $N > 2$ usually system has multiple solutions. EPnP uses relinearization second time, introducing unknowns $\delta_s = \gamma_i \gamma_j$ and equations $\gamma_i \gamma_j = \gamma_k \gamma_l$, which is $(\beta_{i1}\beta_{i2})(\beta_{i3}\beta_{i4}) = (\beta_{i1}\beta_{i3})(\beta_{i2}\beta_{i4})$.

OPnP Algorithm

Constraints are:

$$\lambda_i \tilde{\mathbf{x}} = \mathtt{R}\mathbf{X}_i + \mathbf{t}. \tag{14}$$

Divide by avg depth $\bar{\lambda} = \frac{1}{n}\sum_i \lambda_i$ and denote $\hat{\mathtt{R}} = (\bar{\lambda})^{-1}\mathtt{R}$, $\hat{\mathbf{t}} = (\bar{\lambda})^{-1}\mathbf{t}$, $\hat{\lambda}_i = \frac{\lambda_i}{\bar{\lambda}}$:

$$\hat{\lambda}_i \tilde{\mathbf{x}} = \hat{\mathtt{R}}\mathbf{X}_i + \hat{\mathbf{t}}. \tag{15}$$

Summing up equation triplets, get

$$\hat{t}^{(3)} = n(1 - \hat{\mathbf{r}}_3^T \bar{\mathbf{X}}), \quad \bar{\mathbf{X}} = \frac{1}{n}\sum \mathbf{X}_i.$$

Algorithm OPnP (2)

Parameterize $\hat{\mathtt{R}}$ using non-unit quaternion $\mathbf{q} = (a,\ b,\ c,\ d)^T$:

$$\hat{\mathtt{R}}(\mathbf{q}) = \begin{pmatrix} a^2+b^2-c^2-d^2 & 2bc-2ad & 2bd+2ac \\ 2bc+2ad & a^2-b^2+c^2-d^2 & 2cd-2ab \\ 2bd-2ac & 2cd+2ab & a^2-b^2-c^2+d^2 \end{pmatrix}.$$

Write equations w.r.t. vectorized rotation matrix $\hat{\mathbf{r}}(\mathbf{q})$ и $\hat{\mathbf{t}}^{(1,2)}$:

$$\mathrm{E}_{\mathrm{points}}(\hat{\mathbf{r}}(\mathbf{q}), \hat{\mathbf{t}}) = \|\mathtt{G}_{\mathrm{p}}\hat{\mathbf{r}}(\mathbf{q}) + \mathtt{H}_{\mathrm{p}}\hat{\mathbf{t}}^{(1,2)} + \mathbf{k}_{\mathbf{p}}\|^2 \ \to \min, \tag{16}$$

for known $\mathtt{G}_{\mathrm{p}}, \mathtt{H}_{\mathrm{p}}$ and $\mathbf{k}_{\mathbf{p}}$.

$$\nabla_{\hat{\mathbf{q}}}\mathrm{E}_{\mathrm{points}} = 0,$$

$$\nabla_{\hat{\mathbf{t}}}\mathrm{E}_{\mathrm{points}} = 0.$$

Algorithm OPnP (3)

From $\nabla_{\hat{\mathbf{t}}} \mathrm{E}_{\mathrm{points}} = 0$:

$$\mathtt{H}_{\mathrm{p}}^{\top}(\mathtt{G}_{\mathrm{p}}\hat{\mathbf{r}} + \mathtt{H}_{\mathrm{p}}\hat{\mathbf{t}}^{(1,2)} + \mathbf{k}_{\mathrm{p}}) = 0 \quad \Longrightarrow \quad \hat{\mathbf{t}}^{(1,2)} = \mathtt{P}\hat{\mathbf{r}} + \mathbf{u}, \tag{17}$$

$$\mathtt{P} = -(\mathtt{H}_{\mathrm{p}}^{\top}\mathtt{H}_{\mathrm{p}})^{-1}(\mathtt{H}_{\mathrm{p}}^{\top}\mathtt{G}_{\mathrm{p}}), \;\; \mathbf{u} = -(\mathtt{H}_{\mathrm{p}}^{\top}\mathtt{H}_{\mathrm{p}})^{-1}\mathtt{H}_{\mathrm{p}}^{\top}\mathbf{k}_{\mathrm{p}}. \tag{18}$$

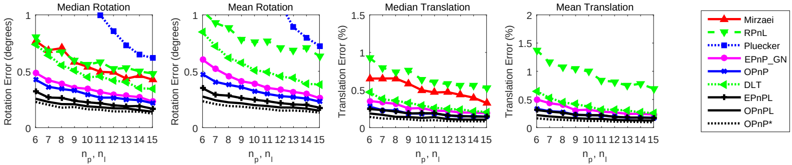
From $\nabla_{\mathbf{q}} \mathrm{E}_{\mathrm{points}} = 0$, for the derivative w.r.t. first quaternion component $\mathbf{q} = (a, \ldots)$:

$$\frac{\partial \hat{\mathbf{r}}}{\partial a}\mathtt{G}_{\mathrm{p}}^{\top}(\mathtt{G}_{\mathrm{p}}\hat{\mathbf{r}} + \mathtt{H}_{\mathrm{p}}\hat{\mathbf{t}}^{(1,2)} + \mathbf{k}_{\mathrm{p}}) = 0. \tag{19}$$

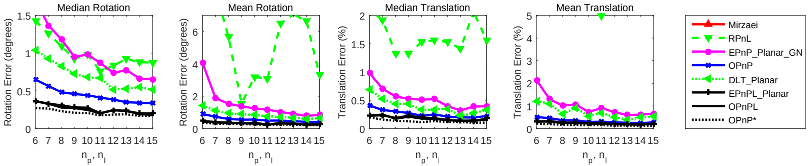
Remains to solve a system of 4 polyomial equations (19) of deg 3.

OPnPL, EPnPL

(a) General case: PnP (n_p), PnL(n_l), PnPL (n_p, n_l)

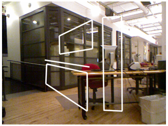
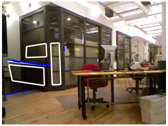
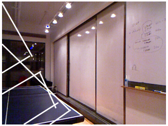
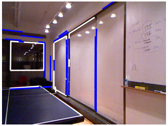
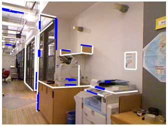
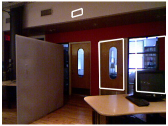
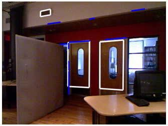


(b) COplanar points and lines: PnP (n_p), PnL(n_l), PnPL (n_p, n_l)

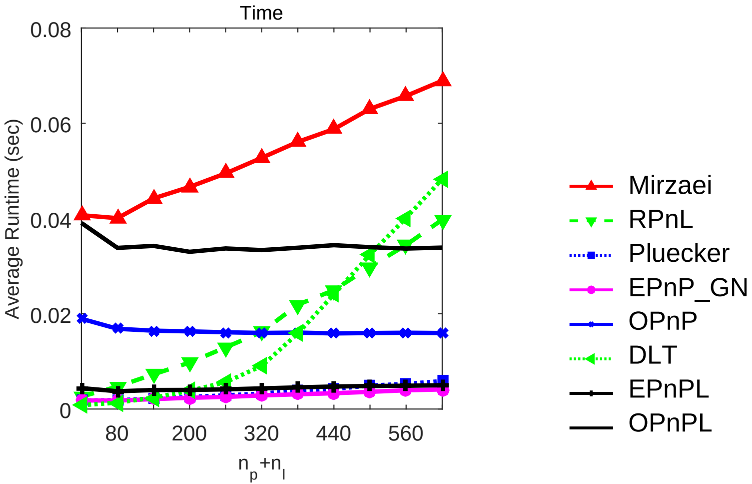


Camera pose from points and lines. Accuracy w.r.t. feature number.

PnPL using NYU2 dataset

OPnP (pt)	OPnPL (lin)/(pt+lin)	OPnP (pt)	OPnPL (lin)/(pt+lin)
$(21.3, 100.0)$	$(9.3, 30.5)/(9.5, 28.2)$	$(61.5, 605.4)$	$(0.4, 6.0)/(0.3, 5.7)$
			
$(1.3, 33.8)$	$(0.6, 9.1)/(0.2, 2.6)$	$(11.5, 100.0)$	$(2.0, 71.8)/(1.3, 30.1)$
			

Time for big n, PnPL problem



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Descriptors

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Алгоритмы PnP

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Datasets

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Conclusion

Thank you for coming!
Thanks for this wonderful opportunity, and Happy New Year!